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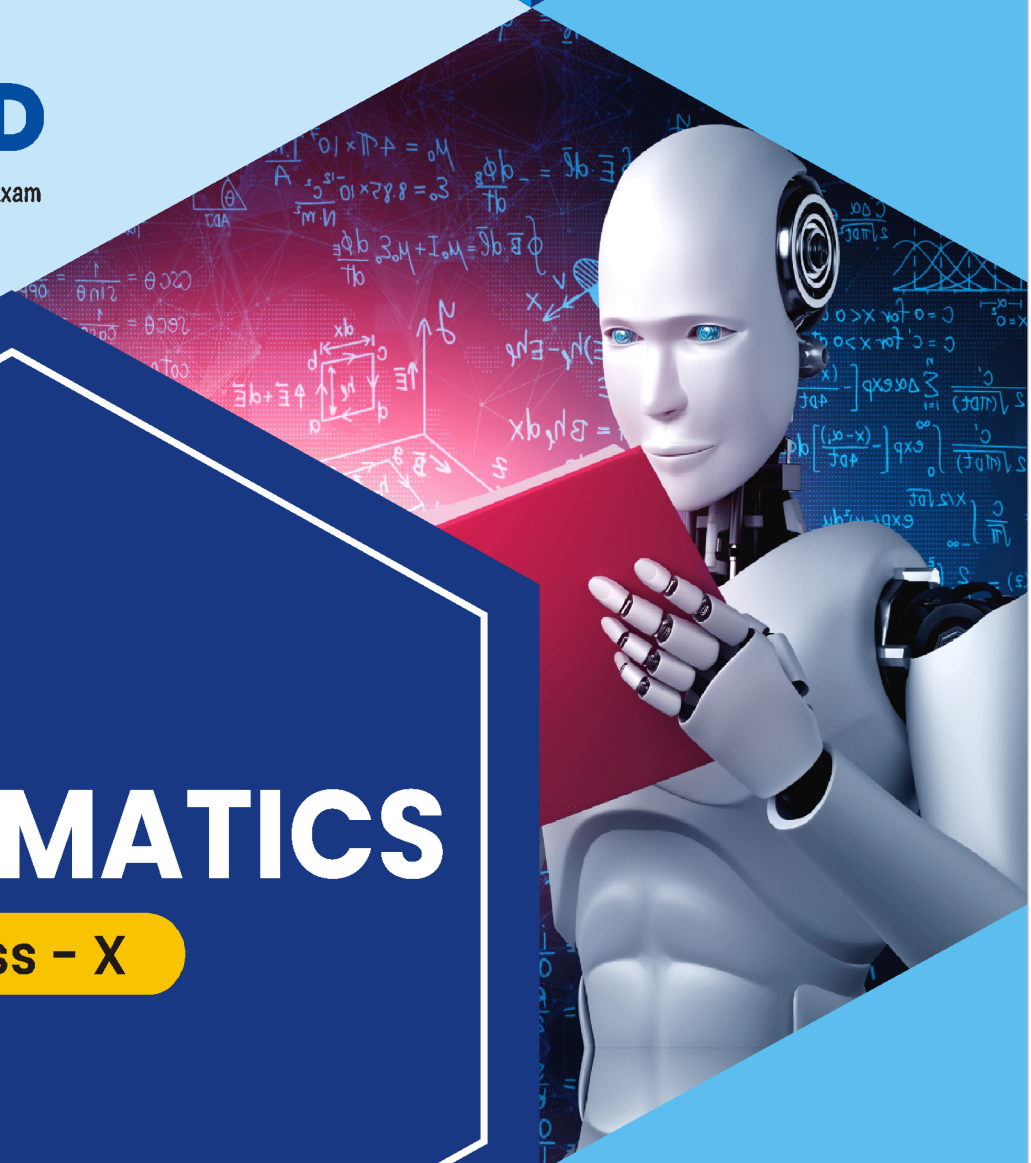

MATHEMATICS

Class - X

MATRIX

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Few words for the Readers

Dear Reader,

"*Matrix Olympiad is established to encourage school students to go a step further than their regular studies, and get a chance and exposure to competition on a wide scale. It also helps students enhance their learning of basic cognitive skills and deeper knowledge of subjects like Science, Mathematics, English, Mental Ability, Social Studies.“Matrix Olympiad helps students nurture their minds for higher targets of tomorrow and enables them to study School for JEE, NEET, CLAT, NDA, Olympiads , NSEJS, NTSE , STSE etc.*"

The above thought has been our guiding principle while designing and collating the study material for **Matrix Olympiad** . And hence, we hope that this particular material will be helpful towards your preparation for **Matrix Olympiad**.

Our team at **MATRIX** has put in their best efforts for making this particular module interesting and relevant for you. Additional efforts have been made to ensure that the content is easy to understand and error free to the extent possible. However, there might remain some inadvertent errors in answer keys and theoretical portion and we would welcome your valuable feedback regarding the same.
If there are any suggestions for corrections, please write to us at **smd@matrixacademy.co.in** and we would be highly grateful.

Finally, we would like to end this message by a famous quote by Ernest Hemingway - "*There is no friend as loyal as a book*." So, please give your study material the time and attention it deserves, and it will surely help you reach newer heights in your fight with competition examinations.

With love and best wishes !
Team MATRIX

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INTRODUCTION

A number that is rational or irrational is called real number, for example 2, $\frac{1}{3}$, –1.5, $\sqrt{3}$, π. The set of real numbers is denoted by R. The positive real number a corresponds to the point whose distance from origin is a units measured in the positive direction, and the negative number b corresponds to the point whose distance from the origin is b units measured in the negative direction. The number zero corresponds to the origin. The line is called a number line or real line.

Focus Point

(i) The numbers, which can be expressed in the form p/q. where p and q are integers and $q \neq 0$ are called rational numbers. Here p is the numerator and q is the denominator when expressed as decimal, a rational number is either terminating or non-terminating repeating.

(ii) the number which cannot be expressed in the form p/q, where p and q are integers and $q \neq 0$ is called an irrational number. When expressed as decimal, an irrational number is a non-terminating, non-repeating decimal number.

1. EUCLID'S DIVISION LEMMA OR DIVISION LEMMA

Let us observe the process of long division of a positive integer a by another positive integer b. Let us take a pair of integer (37, 5), i.e., 37 is divided by 5 and let a = 37, b = 5

Division of 37 by 5 gives quotient equal to 7 and remainder 2. It helps us to write the following relation of equality:

$37 = 5 \times 7 + 2$

In the above relation, we have expressed 37 as the sum of two positive integers.

⇒ So, we can conclude that for pair of positive integers a and b, there exist a pair of unique integers q and r such that $a = b \times q + r$ where $0 \le r < b$.

Here q is the quotient and r is the remainder obtained when a is divided by b. Some facts about the quotient and the remainder are given below:

(i) Quotient q is a positive integer when a is greater than or equal to b.

(ii) Quotient q is 0 when a is less than b.

(iii) Remainder r is a whole number whose value is less than b i.e., $0 \le r < b$.

Let us write some more relations of the form $a = b \times q + r$, $0 \le r < b$.

(i) $47 = 9 \times q + r$ (ii) $83 = 17 \times q + r$ (iii) $75 = 15 \times q + r$ (iv) $7 = 37 \times q + r$

Observe that in (i) q = 5, r = 2, in (ii) q = 4, r = 15, in (iii) q = 5, r = 0, in (iv) q = 0, r = 7

We have understood that the value of q and r in each of the above cases is a unique integer and we can write the values of q and r uniquely. From the above observations, we can now state the Euclid's division algorithm.

2. EUCLID'S DIVISION ALGORITHM

To obtain the H.C.F. of two positive integers, say a and b with a > b, we use Euclid's Division Algorithm.

Step 1 **:** Apply Euclid's division Lemma, to a and b to find whole numbers, q and r such that $a = b \times q + r$, $0 \le r < b$.

Step 2 **:** If r = 0, b is the H.C.F. of a and b. If $r \neq 0$, apply the division lemma to b and r.

Step 3 **:** Continue the process till the remainder is zero. The divisor at this stage will be the required H.C.F.

For example : We calculate the H.C.F. of 595 and 721 as follows:

$721 = 1 \times 595 + 126$ $\quad$ $595 = 4 \times 126 + 91$

$126 = 1 \times 91 + 35$ $\quad$ $91 = 2 \times 35 + 21$

$35 = 1 \times 21 + 14$ $\quad$ $21 = 1 \times 14 + 7$

$14 = 2 \times 7 + 0$

Thus, the remainder has become zero when divisor is 7. Therefore, H.C.F. (595, 721) = 7.

Example 1

Show that every positive even integer is of the form 2q and every positive odd integer is of the form 2q + 1 for some integer q.

Solution :

Let a be a given positive integer.

On dividing a by 2, let q be the quotient and r be the remainder.

Then, by Euclid's Lemma, we have

$a = 2q + r$, where $0 \le r < 2$

$\Rightarrow$ $a = 2q + r$, where r = 0 or r = 1

$\Rightarrow$ $a = 2q$ or $a = 2q + 1$

When a = 2q for some integer q, then clearly a is even.

Also an integer can be either even or odd.

Hence, an odd integer is of the form 2q + 1 for some integer q.

Thus, every positive even integer is of the form 2q and every positive odd integer is of the form 2q + 1.

3. FUNDAMENTAL THEOREM OF ARITHMETIC

Statement : Every composite number can be expressed (factorised) as a product of primes and this factorization is unique, apart from the order in which the prime factors occur.

4. H.C.F. AND L.C.M. USING PRIME FACTORIZATION METHOD

H.C.F. = Product of the smaller power of each common prime factor involved in the numbers.

L.C.M. = Product of the greatest power of each prime factor involved in the numbers.

Note : For any two positive integers a and b, H.C.F. (a, b) × L.C.M. (a, b) = a × b.

Focus Point

For three numbers (a, b and c)

(i) H.C.F. (a, b, c) × L.C.M. (a, b, c) $\neq$ a × b × c, where a, b, c, are positive integers.

(ii) $$\text{L.C.M.}(a,b,c) = \frac{a \times b \times c \times \text{H.C.F.}(a,b,c)}{\text{H.C.F.}(a,b) \times \text{H.C.F.}(b,c) \times \text{H.C.F.}(a,c)}$$

(iii) $$\text{H.C.F.}(a,b,c) = \frac{a \times b \times c \times \text{L.C.M.}(a,b,c)}{\text{L.C.M.}(a,b) \times \text{L.C.M.}(b,c) \times \text{L.C.M.}(a,c)}$$

Example 2

Consider the numbers 4^n, where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero.

Solution :

If the number 4^n, for any n, has to end with the digit zero, then it would be divisible by 5. That is, the prime factorisation of 4^n would contain the prime number 5. This is not possible because $4^n = (2)^{2n}$, so the only prime in the factorization of 4^n is 2. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 4^n. So, there is no natural number n for which 4^n ends with the digit zero.

Example 3

Write the prime factorization of the number 27300. In the factorization find (i) the total number of primes and (ii) the total number of distinct primes.

Solution :

2	27300
2	13650
3	6825
5	2275
5	455
7	91
	13

Therefore, the required factorization is

$$27300 = 2 \times 2 \times 3 \times 5 \times 5 \times 7 \times 13$$

(i) The total number of primes in the above factorization are 7

(ii) The total number of distinct primes are 5 which are 2, 3, 5, 7 and 13.

Example 4

Find the L.C.M. and H.C.F. of 9, 117 and 729 by the prime factorization method.

Solution :

The prime factorisation of 9, 117 and 729 gives:

$$9 = 3 \times 3,\ 117 = 3 \times 3 \times 13,$$

$$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$\Rightarrow$ $9 = 3^2,\ 117 = 3^2 \times 13,\ 729 = 3^6$

H.C.F. (9, 117, 729) $= 3^2 = 9$

L.C.M. (9, 117, 729) $= 3^6 \times 13$

$= 729 \times 13 = 9477.$

Example 5

Find the H.C.F. of (8, 648) by prime factorization and hence find the L.C.M.

Solution :

The prime factorization of 8 and 648 gives:

$$8 = 2 \times 2 \times 2,\ 648 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$$

$\Rightarrow$ $8 = 2^3,\ 648 = 2^3 \times 3^4$; H.C.F. $(8, 648) = 2^3 = 8$

$$\text{L.C.M.}(8, 648) = \frac{\text{Product of two numbers } (8, 648)}{\text{HCF } (8, 648)}$$

$$= \frac{8 \times 648}{8} = 648$$

5. IRRATIONAL NUMBERS

A number is called irrational, if it cannot be written in the form p/q, where p and q are integers and $q \neq 0$ or a decimal number which is neither terminating nor recurring.

For example : $\sqrt{2}, \sqrt{3}, \pi$, etc.

5.1 THEOREM

If a prime number p divides a^2, then p divides a, where a is a positive integer.

Proof : Let prime factorization of $a = p_1 \times p_2 \times \ldots\ldots p_n$ where $p_1, p_2, \ldots\ldots, p_n$ are primes

$\Rightarrow \quad a^2 = \{p_1 \times p_2 \times ..... \times p_n\}^2 = p_1^2 \times p_2^2 \times ..... \times p_n^2$

Now if p divides $a^2 \Rightarrow p$ is a factor of $p_1^2 \times p_2^2 \times ..... \times p_n^2$

We know that primes in the factorization $p_1^2 \times p_2^2 \times ..... \times p_n^2$ are unique

[Fundamental theorem of Arithmetic]

$\Rightarrow$ p is a prime which is one out of the primes $p_1, p_2, ...p_n$.

Suppose $p = p_k$ for some value of k from 1 to n. Now, p_k divides $\{p_1 \times p_2 \times .....p_n\}$

$\Rightarrow$ p_k divides a

$\Rightarrow$ p divides a $[\because p_k = p]$

Hence, the theorem is proved.

Example 6

Show that $\sqrt{7}$ is irrational.

Solution :

Let us assume to the contrary, that $\sqrt{7}$ is rational.

So, we can find integers r and s ($s \neq 0$) such that $\sqrt{7} = \frac{r}{s}$.

Suppose r and s have a common factor other than

1. Then, we divide by the common factor to get $\sqrt{7} = \frac{a}{b}$, where a and b are co-prime.

So, $b\sqrt{7} = a$

Squaring both sides and rearranging, we get $7b^2 = a^2$. Therefore, 7 divides a^2.

Now, it follows that 7 divides a.

[By Theorem 6.1]

So, we can write a = 7c for some integer c. Substituting a, we get $\sqrt{7}b = 7c$

Squaring both sides, we get

$7b^2 = 49c^2 \quad \Rightarrow b^2 = 7c^2$

$\Rightarrow$ 7 divides b^2 $\quad \Rightarrow$ 7 divides b

$\Rightarrow$ a and b have 7 as a common factor. But this contradicts the fact that a and b have no common factor other than 1.

This contradiction has arisen because of our incorrect assumption that $\sqrt{7}$ is rational. So, we conclude that $\sqrt{7}$ is irrational.

6. RATIONAL NUMBERS AND THEIR DECIMAL EXPANSIONS

We know that the decimal expansions of the rational numbers of the form $\frac{p}{q} (q \neq 0)$ are either terminating or non-terminating repeating (i. e., recurring).

Let us consider some terminating decimal expansions.

(i) 0.25 (ii) 0.773

Now, we have

(i) $0.25 = \frac{25}{100} = \frac{1}{4} = \frac{1}{2^2 \times 5^0}$ (ii) $0.773 = \frac{773}{1000} = \frac{773}{2 \times 2 \times 2 \times 5 \times 5 \times 5} = \frac{773}{2^3 \times 5^3}$

In each case, the terminating decimal expansion reduces to rational number of the form p/q, where p and q are co-prime and the prime factorization of q is of the form $2^m \times 5^n$ (m and n are non-negative integers).

6.1 THEOREM

Let x be rational number whose decimal expansion terminates. Then, x can be expressed in the form p/q, where p and q co-prime and the prime factorization of q is of the form $2^m \times 5^n$, where m, n are non-negative integers.

Focus Point

- The sum of a rational number and an irrational number is always irrational.
- The product of a rational number ($\neq 0$) and an irrational number is always irrational.
- The sum of two irrational numbers is not always an irrational number.
- The product of two irrational numbers is not always an irrational number.

6.2 THEOREM

If a rational number is of the form $\frac{p}{q} (q \neq 0)$ with prime factorization of q of the form $2^m \times 5^n$, where m, n are non-negative integers, then p/q has a terminating decimal.

Proof : Let p/q be a rational number in the lowest form such that the prime factorization of q is of the form $2^m \times 5^n$, where m, n are non-negative integers.

We have the following cases :

Case - I : When m = n

In this case, we have $\frac{p}{q} = \frac{p}{2^m \times 5^n} = \frac{p}{2^m \times 5^m} = \frac{p}{(10)^m}$

Case - II : When m > n

In this case, we have n = m + a, where a is a positive integer.

$\therefore \frac{p}{q} = \frac{p}{2^m \times 5^n} = \frac{p \times 5^a}{2^m \times 5^{n+a}} = \frac{p \times 5^a}{2^m \times 5^m} = \frac{p \times 5^a}{(2 \times 5)^m} = \frac{c}{10^m}$, where $c = p \times 5^a$

Case - III : When m < n

In this case, we have n = m + a, where a is a positive integer.

$\therefore \frac{p}{q} = \frac{p}{2^m \times 5^n} = \frac{p \times 2^a}{2^{m+a} \times 5^n} = \frac{p \times 2^a}{2^n \times 5^n} = \frac{p \times 2^a}{(2 \times 5)^n} = \frac{c}{10^n}$, where $c = p \times 2^a$

Thus, a rational number whose denominator is of the form $2^m \times 5^n$, where m, n are non-negative integers, can be converted to an equivalent rational number of the form c/d, where d is a power of 10.

For example,

(i) $\frac{7}{8}=\frac{7}{2^3}=\frac{7\times5^3}{2^3\times5^3}=\frac{7\times125}{(2\times5)^3}=\frac{875}{10^3}$ (ii) $\frac{2139}{1250}=\frac{2139}{2^1\times5^4}=\frac{2139\times2^3}{2^4\times5^4}=\frac{2139\times8}{(2\times5)^4}=\frac{17112}{10^4}$

6.3 THEOREM

Let x = p/q be a rational number, such that the prime factorization of q is not of the form $2^m \times 5^n$, where m, n are non-negative integers. Then x has decimal expansion which is non-terminating repeating (recurring).

Example 7

Express 0.737373..... in the form $\frac{p}{q}$. Also find the prime factorization of q when p and q are co-prime.

Solution :

Let a = 0.737373.... = $0.\overline{73}$ .....(i)

$\Rightarrow$ 100a = 73.737373...= $73.\overline{73}$ .....(ii)

On subtracting (i) from (ii), we get

$\Rightarrow$ 99a = 73 $\Rightarrow$ $a=\frac{73}{99}=\frac{p}{q}$

$\Rightarrow$ p = 73 and q = 99 which are co-prime

Here, $q = 3^2 \times 11$

Example 8

Express $0.2\overline{54}$ as a fraction in simplest form.

Solution :

Let $x = 0.2\overline{54}$ ....(i)

then x = 2545454 .....

$\therefore$ 100x = 25.4545 ....(ii)

On subtracting (i) from (ii), we get

$99x = 25.2 \Rightarrow x=\frac{252}{990}=\frac{14}{55}$. Thus, $0.2\overline{54}=\frac{14}{55}$

Focus Point

(i) $a^n + b^n$ is always divisible by a + b when 'n' is odd, for example $7^{11} + 6^{11}$ is divisible by 13.

(ii) $a^n - b^n$ is always divisible by a – b when 'n' is odd, for example $13^{11} - 2^{11}$ is divisible by 11.

(iii) $a^n - b^n$ is always divisible by a – b and a + b when 'n' is even, for example $23^{50} - 3^{50}$ is divisible by both 20 and 26.

SOLVED EXAMPLES

SE. 1

Find the H.C.F and L.C.M of 30, 72 and 432 using the prime facorization method.

Ans. By prime factorization, we get

2	30
3	15
	5

2	72
2	36
2	18
3	9
	3

2	432
2	216
2	108
2	54
3	27
3	9
	3

$\therefore$ $30 = 2 \times 3 \times 5$

$72 = 2^3 \times 3^2$

$432 = 2^4 \times 3^3$

$\therefore$ H.C.F. (30, 72, 432) = $(2 \times 3) = 6$

L.C.M. (30, 72, 432) = $(2^4 \times 3^3 \times 5)$

$= (16 \times 27 \times 5) = 2160$

SE. 2

Using Euclid's division algorithm, find H.C.F. of 274170 and 17017.

Ans. By Euclid's Division Lemma, we have $274170 = 17017 \times 16 + 1898$

We apply Euclid Division Lemma on the divisor 17017 and the remainder 1898.

$17017 = 1898 \times 8 + 1833$

Similarly we have

$1898 = 1833 \times 1 + 65$

$1833 = 65 \times 28 + 13$

$65 = 13 \times 5 + 0$

Thus remainder is 0 when divisor is 13.

So by the Euclid's Division Algorithm H.C.F. (274170, 17017) = 13

SE. 3

In the following equations, find which variables x, y, z etc. represent rational or irrational numbers.

(i) $x^2 = 5$ (ii) $y^2 = 9$

(iii) $z^2 = 0.04$ (iv) $u^2 = 17/4$

(v) $v^2 = 3$ (vi) $w^3 = 27$

(viii) $t^2 = 0.4$

Ans. (i) $x^2 = 5 \Rightarrow x = \pm\sqrt{5} = \pm\ 2.2360679...$ which are irrational numbers.

(ii) $y^2 = 9 \Rightarrow \pm 3$ which are rational numbers.

(iii) $z^2 = 0.04 \Rightarrow z = \pm\ 0.2$ which are rational numbers.

(iv) $u^2 = 17/4 \Rightarrow u = \pm\dfrac{\sqrt{17}}{2} = \pm 2.0615528...$

which are irrational numbers.

(v) $v^2 = 3 \Rightarrow v = \pm\sqrt{3} = \pm 1.7320508...$

which are irrational numbers

(vi) $w^3 = 27 \Rightarrow w = 3$ which is a rational number

(viii) $t^2 = 0.4 \Rightarrow t = \pm\ 0.63245...$ which are irrational numbers.

SE. 4

Give example of two irrational numbers, whose

(i) product is an irrational number

(ii) product is a rational number

(iii) quotient is a rational number

(iv) quotient is an irrational number

Ans. (i) Let $\sqrt{2}$ and $\sqrt{3}$ be two irrational numbers.

Their product $= (\sqrt{2}) \times (\sqrt{3}) = (\sqrt{6})$, which is an irrational number

(ii) Let $(2+\sqrt{3})$ and $(2-\sqrt{3})$ be two irrational numbers.

Their product = $(2+\sqrt{3})\ (2-\sqrt{3}) = 4 - 3 = 1$, which is a rational number.

(iii) Let $3+3\sqrt{2}$ and $1+\sqrt{2}$ be irrational numbers.

Their quotient $= \dfrac{3+3\sqrt{2}}{1+\sqrt{2}} = \dfrac{3(1+\sqrt{2})}{1+\sqrt{2}} = 3$, which is a rational number.

(iv) Let $\sqrt{10}-\sqrt{5}$ and $\sqrt{2}-1$ be two irrational numbers. Their quotient

$$= \frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}-1} = \frac{\sqrt{5}(\sqrt{2}-1)}{(\sqrt{2}-1)}$$

which is an irrational number.

SE. 5

Prove that if x and y are odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

Ans. We know that any odd positive integer is of the form $2q + 1$ for some integer q.

So, let $x = 2m + 1$ and $y = 2n + 1$ for some integer m and n.

$\therefore\ x^2 + y^2 = (2m + 1)^2 + (2n + 1)^2$

$\Rightarrow\ x^2 + y^2 = 4(m^2 + n^2) + 4(m + n) + 2$

$\Rightarrow\ x^2 + y^2 = 4\{(m^2 + n^2) + (m + n)\} + 2$

$\Rightarrow\ x^2 + y^2 = 4q + 2$, where $q = (m^2 + n^2) + (m + n)$

$\Rightarrow\ x^2 + y^2$ is even and leaves remainder 2 when divided by 4

$\Rightarrow\ x^2 + y^2$ is even but not divisible by 4.

SE. 6

Express $0.3\overline{6}$ as a fraction in simplest form.

Ans. Let $x = 0.3\overline{6}$. Then $x = 0.366...$

$\therefore\ 10x = 3.666$

On subtracting (i) from (ii), we get

$9x = 3.3 \Rightarrow x = \frac{33}{90} = \frac{11}{30}$.

Hence, $x = 0.3\overline{6} = \frac{11}{30}$

SE. 7

Show that one and only one out of n, n + 3, n + 6 or n + 9 is divisible by 4.

Ans. Let us apply the division algorithm with a = n and b = 4, we get

$n = 4q + r, \quad 0 \le r < 4$

$\Rightarrow$ $n = 4q + r$, where r can have one and only one value of 0, 1, 2 or 3.

$\Rightarrow$ $n = 4q$ or $4q + 1$, or $4q + 2$, or $4q + 3$ and one and only one of these above possiblities can happen.

(i) If $n = 4q$, then n is divisible by 4

(ii) If $n = 4q + 1$, then $n + 3 = 4q + 4 = 4(q + 1)$, and therefore n + 3 is divisible by 4.

(iii) If $n = 4q + 2$, then $n + 6 = 4q + 8 = 4(q + 2)$, therefore n + 6 is divisible by 4.

(iv) If $n = 4q + 3$, then $n + 9 = 4q + 12 = 4(q + 3)$, therefore n + 9 is divisible by 4.

SE. 8

For any positive real number x, prove that there exist an irrational number y such that $0 < y < x$.

Ans. If x is irrational, then $y = \frac{x}{2}$ is also an irrational number such that $0 < y < x$.

If x is rational, then $y = \frac{x}{\sqrt{2}}$ is also an irrational number such that $\frac{x}{\sqrt{2}} < x$ as $\sqrt{2} > 1$.

$\therefore\ y = \frac{x}{\sqrt{2}}$ is an irrational number such that $0 < y < x$.

SE. 9

If d is the H.C.F of 56 and 72, find x, y satisfying d = 56x + 72y. Also, show that x and y are not unique.

Ans. Applying Euclid's division lemmna to 56 and 72, we get $72 = 56 \times 1 + 16$ ....(i)

Since the remainder $16 \neq 0$.

So, consider th divisor 56 and the remainder 16 and apply division lemma to get

$56 = 16 \times 3 + 8$ ....(ii)

Similarly, we get

$16 = 8 \times 2 + 0$ ....(iii)

We observe that the remainder at this state is zero.

So, H.C.F (56, 72) = 8

From (ii), we have

$8 = 56 - 16 \times 3$

$\Rightarrow\ 8 = 56 - (72 - 56 \times 1) \times 3$

[$\because 16 = 72 - 56 \times 1$, from (i)]

$\Rightarrow\ 8 = 56 - 3 \times 72 + 56 \times 3$

$\Rightarrow\ 8 = 56 \times 4 + (-3) \times 72$

$\therefore\ x = 4$ and $y = -3$

Now, $8 = 56 \times 4 + (-3) \times 72$

$8 = 56 \times 4 + (-3) \times 72 - 56 \times 72 + 56 \times 72$

$\Rightarrow \quad 8 = 56 \times 4 - 56 \times 72 + (-3) \times 72 + 56 \times 72$

$\Rightarrow \quad 8 = 56 \times (4 - 72) + \{(-3) + 56\} \times 72$

$\Rightarrow \quad 8 = 56 \times (-68) + \{53\} \times 72$

$\therefore \quad x = -68$ and $y = 53$

Hence, x and y are not unique.

SE. 10

Show that $\sqrt[3]{6}$ is not a rational number.

Ans. Let us assume that $\sqrt[3]{6}$ is a rational number.

Let $\sqrt[3]{6} = \frac{p}{q}, q \neq 0 \Rightarrow \frac{p^3}{q^3} = 6$ ....(1)

[Taking cube on both sides]

Now, $1^3 = 1$ and $2^3 = 8$. Also, $1 < 6 < 8$

$\Rightarrow 1 < \frac{p^3}{q^3} < 8 \qquad \Rightarrow 1 < \frac{p}{q} < 2$

From (1), $\frac{p^3}{q^3} = 6 \qquad \Rightarrow 6q^2 = \frac{p^3}{q}$ ....(2)

As q is an integer $\Rightarrow 6q^2$ is an integer ....(3)

Since p and q have no common factor

$\Rightarrow p^3$ and q will have no common factor.

$\Rightarrow \frac{p^3}{q}$ is a fraction ....(4)

From (2), (3) and (4), we get

Integer = Fraction

Thus, out assumption is wrong.

Hence, $\sqrt[3]{6}$ is not a rational number.

SE. 11

Show that there is no positive integer n for which $\sqrt{n-1} + \sqrt{n+1}$ is rational.

Ans. If possible, let there be positive integer n for which $\sqrt{n-1} + \sqrt{n+1}$ is rational and equal to $\frac{a}{b}$ (say), where a, b are positive integers.

Then $\frac{a}{b} = \sqrt{n-1} + \sqrt{n+1}$ ....(i)

$$\frac{b}{a} = \frac{1}{\sqrt{n-1} + \sqrt{n+1}}$$

$$\Rightarrow \frac{b}{a} = \frac{\sqrt{n+1} - \sqrt{n-1}}{\{\sqrt{n+1} + \sqrt{n-1}\}\{\sqrt{n+1} - \sqrt{n-1}\}}$$

$$= \frac{\sqrt{n+1} - \sqrt{n-1}}{(n+1) - (n-1)} = \frac{\sqrt{n+1} - \sqrt{n-1}}{2}$$

$\Rightarrow \quad \frac{2b}{a} = \sqrt{n+1} - \sqrt{n-1}$ ....(ii)

Adding (i) and (ii) & subtracting (ii) from (i), we get

$2\sqrt{n+1} = \frac{a}{b} + \frac{2b}{a}$ and $2\sqrt{n-1} = \frac{a}{b} - \frac{2b}{a}$

$\Rightarrow \quad \sqrt{n+1} = \frac{a^2 + 2b^2}{2ab}$ and $\sqrt{n-1} = \frac{a^2 - 2b^2}{2ab}$

$\therefore \sqrt{n+1}$ and $\sqrt{n-1}$ are rationals.

[$\because$ a, b are integers]

[$\therefore \frac{a^2 + 2b^2}{2ab}$ & $\frac{a^2 - 2b^2}{2ab}$ are rationals]

$\Rightarrow$ (n + 1) and (n – 1) are squares of positive integers.

This is not possible as any two perfect squares differ at least by 3.

Hence, there is no positive integers n for which $(\sqrt{n-1} + \sqrt{n+1})$ is rational.

SE. 12

Prove that $\sqrt{n}$ is not a rational number, if n is not a perfect square.

Ans. Let $\sqrt{n}$ be a rational number

$\therefore \quad \sqrt{n} = \frac{p}{q}$ [p and q are co-prime]

$\Rightarrow \quad n = \frac{p^2}{q^2}$ [Squaring both sides]

$\Rightarrow \quad p^2 = nq^2$ .....(1)

$\Rightarrow$ n divides p^2 $\Rightarrow$ n divides p .....(2)

Let p = nm $\Rightarrow p^2 = n^2m^2$

[Squaring both side]

Putting the value of p^2 in (1), we get

$n^2m^2 = nq^2$

$\Rightarrow \quad q^2 = nm^2$

$\Rightarrow$ n divides q^2 $\Rightarrow$ n divides q ....(3)

From (2), n divides p and from (3) n divides q. It means n is a common factor of both p and q.

This contradicts the assumption that p and q are co-prime. So, our supposition is wrong.

Hence, $\sqrt{n}$ cannot be a rational number.

SE. 13

For any positive integer n, prove that $n^3 - n$ is divisible by 6.

Ans. $n^3 - n = n(n^2 - 1)$

$= n(n + 1)(n - 1) = (n - 1)\, n(n + 1)$

= product of three consecutive positive integers.

Now, we have to show that the product of three consecutive positive integers is divisible by 6.

We knwo that any positive integer a is of the form 3q, 3q + 1 or 3q + 2 for some integer q.

Let a, a + 1, a + 2 be any three consecutive integers.

Case-I : If a = 3q.

$a(a + 1)(a + 2) = 3q(3q + 1)(3q + 2)$

$= 3q(2r) = 6qr$, which is divisible by 6.

($\because$ Product of two consecutive integers (3q + 1) and (3q + 2) is an even integer, say 2r)

Case-II : If a = 3q + 1

$\therefore \quad a(a + 1)(a + 2) = (3q + 1)(3q + 2)(3q + 3)$

$= (2r)(3)(q + 1) = 6r(q + 1)$,

which is divisible by 6.

Case-III

$\therefore \quad a(a + 1)(a + 2) = (3q + 2)(3q + 3)(3q + 4)$

= multiple of 6 for every q

= 6r(say),

which is divisible by 6.

Hence, the product of three consecutive integers is divisible by 6.

SE. 14

Find the H.C.F. and L.C.M. of 25152 and 12156 by using by Fundamental Theorem of Arithmetic.

Ans. The prime factors of 25152 and 12156 are given below.

$25152 = 2^6 \times 3 \times 131$

$12156 = 2^2 \times 3 \times 1013$

$\therefore$ H.C.F. (25152, 12156) = Product of the smallest power of each common prime factors in the number $= 2^2 \times 3 = 12$

We know that

H.C.F. (25152, 12156) × L.C.M. (25152, 12156)

$= 25152 \times 12156$

$\Rightarrow 12 \times$ L.C.M. (25152, 12156) $= 25152 \times 12156$

$\Rightarrow$ L.C.M. (25152, 12156) $= \frac{25152 \times 12156}{12}$

$= 25152 \times 1013 = 25478976$

EXERCISE – I

ONLY ONE CORRECT TYPE

1. Which of the following is always true ?

(A) The rationalising factor of a number is unique

(B) The sum of two distinct irrational numbers is rational

(C) The product of two distinct irrational numbers is irrational

(D) None of these

2. If n is an odd natural number, $3^{2n} + 2^{2n}$ is always divisible by

(A) 13 (B) 5

(C) 17 (D) 19

3. Which of the following values are even ?

(a) 21 + 18 + 9 + 2 + 19

(b) 34 × 28 × 37 × 94 × 12712

(c) 33 × 35 × 37 × 39 × 41 × 43

(d) 11 × 11 × 11 × 11 × 11 × ....

(e) 1^{10}

(f) 39 – 24

(A) a,b,c (B) d,e,f

(C) b (D) a,b,d,e

4. Find the unit digit in the expansion of $(44)^{44} + (55)^{55} + (88)^{88}$.

(A) 7 (B) 5

(C) 4 (D) 3

5. Find the digit in the units place of $(676)^{99}$.

(A) 9 (B) 2

(C) 4 (D) 6

6. The greatest five digit number exactly divisible by 9 and 13 is

(A) 99945 (B) 99918

(C) 99964 (D) 99972

7. If the number 2345p60q is exactly divisible by 3 and 5, then the maximum value of p + q is

(A) 12 (B) 13

(C) 14 (D) 15

8. A rational number can be expressed as a terminating decimal if the denominator has factors

(A) 2 or 5 (B) 2 ,3 or 5

(C) 3 or 5 (D) None of these

9. The value of $23.\overline{43} + 5.\overline{2}$ is

(A) $\frac{2395}{990}$ (B) $\frac{2527}{99}$

(C) $\frac{5169}{990}$ (D) $\frac{2837}{99}$

10. The product of rational and irrational number is always

(A) Rational (B) Irrational

(C) Both (D) Can't say

11. The least number which when increased by 5 is divisible by each one of 24, 32, 36 and 54, is

(A) 427 (B) 859

(C) 869 (D) 4320

12. The largest four-digit number which when divided by 4, 7 or 13 leaves a remainder of 3 in each case, is

(A) 8739 (B) 9831

(C) 9834 (D) 9893

13. Find the least multiple of 23, which when divided by 18, 21 and 24 leaves remainders 7, 10 and 13 respectively.

(A) 3002 (B) 3013

(C) 3024 (D) 3036

14. The L.C.M. of two numbers is 39780 and their ratio is 13 15 then the numbers are

(A) 273, 315 (B) 2652, 3060

(C) 516, 685 (D) None of these

15. The sum of three non-zero prime numbers is 100. One of them exceeds the other by 36. Find the largest number.

(A) 73 (B) 91

(C) 67 (D) 57

16. Find the least number which when divided by 12, leaves a remainder of 7, when divided by 15, leaves a remainder of 10 and when divided by 16, leaves a remainder of 11.

(A) 115 (B) 235

(C) 247 (D) 475

17. Two numbers are in the ratio of 15 : 11. If their H.C.F. is 13, then numbers will be :

(A) 195 and 143 (B) 190 and 140

(C) 185 and 163 (D) 185 and 143

18. Which of the following is a rational number ?

(A) Sum of $2+\sqrt{3}$ and its inverse

(B) Square root of 18

(C) Square root of $7+4\sqrt{3}$

(D) None of these

19. Three farmers have 490 kg, 588 kg and 882 kg weights of wheat respectively. Find the maximum capacity of a bag so that the wheat can be packed in exact number of bags.

(A) 96 kg (B) 95 kg

(C) 94 kg (D) 98 kg

20. A trader has a basket of eggs. If counted the eggs in pairs one will remain; If counted in 3, two will remain; If counted in 4, 3 will remain; If counted in 5, 4 will ramain; If counted in 6, 5 will remain; If counted in 7, nothing will remain, and the basket cannot accmmodate more than 150 eggs. So, how many eggs were there ?

(A) 117 (B) 123

(C) 119 (D) 121

21. The greatest number of 6 digits exactly divisible by 15, 24 and 36 is

(A) 999998 (B) 999999

(C) 999720 (D) 999724

22. Let x be the greatest number by which if we divide 366, 513 and 324, then in each case the remainder is the same. The sum of digits of x is

(A) 3 (B) 4

(C) 5 (D) 7

23. In a seminar, the number of participants in English, Mathematics and Science are 36, 60 and 84 respectively. Find the minimum number of rooms required if in each room the same number of participants are to be seated and all of them being in the same subject.

(A) 10 (B) 15

(C) 20 (D) 8

24. Find the remainder when $7^{21} + 7^{22} + 7^{23} + 7^{24}$ is divided by 25.

(A) 0 (B) 2

(C) 4 (D) 6

25. If N = 901 × 902 × 903. If N is divided by 25 the remainder is :

(A) 0 (B) 2

(C) 6 (D) 8

PARAGRAPH TYPE

PASSAGE # I

The largest or greatest among common divisors of two or more integers is called the Greatest common Divisor (GCD) or Highest Common Factor (H.C.F.)

26. The largest number which divides 285 and 1249 leaving remainders 9 and 7 respectively, is

(A) 46 (B) 6

(C) 12 (D) 138

27. Find H.C.F. (2002, 2618).

(A) 11 (B) 22

(C) 154 (D) 13

28. Two brands of chocolate are available in packs of 24 and 15 respectively. If I need to buy an equal number of chocolate of both kinds, then what is the least number of boxes of each kind I would need to buy ?

(A) 5, 6 (B) 5, 8

(C) 5, 4 (D) 12, 14

PASSAGE # II

Every composite number can be expressed as a product of primes and this factorization is unique except the order in which prime factor occurs.

29. Express 945 as a product of prime factors.

(A) $3 \times 5^3 \times 7$ (B) $3^2 \times 5^2 \times 7$

(C) $3^3 \times 5 \times 7$ (D) $21 \times 3^2 \times 5$

30. Determine prime factotization of 20570.

(A) $2 \times 5 \times 11^2 \times 17$ (B) $10 \times 11^2 \times 17$

(C) $5 \times 3^4 \times 121$ (D) $17 \times 10^2 \times 11$

31. Determine prime factorization of 205751.

(A) $49 \times 13 \times 19^2$ (B) $7^2 \times 13 \times 17 \times 19$

(C) $7 \times 13 \times 17 \times 91$ (D) $7 \times 13^2 \times 17 \times 19$

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from List - I and List - II are given as options (A), (B), (C) and (D) out of which one is correct.

32. List – II gives H.C.F. for pair given in List – I, match them correctly.

List – I	List – II
(P) 135 and 255	(i) 8
(Q) 196 and 38220	(ii) 51
(R) 255 and 867	(iii) 15
(S) 616 and 32	(iv) 196

(A) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)

(B) (P) → (iii), (Q) → (ii), (R) → (iv), (S) → (i)

(C) (P) → (iii), (Q) → (iv), (R) → (ii), (S) → (i)

(D) (P) → (i), (Q) → (iv), (R) → (iii), (S) → (ii)

33. List – II gives L.C.M. for pair given in List – I, match them correctly.

List – I	**List – II**
(P) 92 and 510	(i) 546
(Q) 306 and 657	(ii) 23460
(R) 54 and 336	(iii) 22338
(S) 6 and 91	(iv) 3024

(A) (P) → (ii), (Q) → (iii), (R) → (iv), (S) → (i)

(B) (P) → (ii), (Q) → (iv), (R) → (iii), (S) → (i)

(C) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)

(D) (P) → (iii), (Q) → (iv), (R) → (ii), (S) → (i)

Space for Notes :

EXERCISE – II

VERY SHORT ANSWER TYPE

1. Define L.C.M. (Least Common Multiple).
2. Define Fundamental Theorem of Arithmetic.
3. If two numbers and their L.C.M. is given, then how we find H.C.F. of the numbers.
4. Find the H.C.F. of $2^3 \times 3^2 \times 5 \times 7^4$, $2^2 \times 3^5 \times 5^2 \times 7^3$, $2^3 \times 5^3 \times 7^2$.
5. Find the H.C.F. of 108, 288 and 360.
6. Which of the following is terminating decimal expansion ?

   (i) $\frac{13}{25}$ (ii) $\frac{9}{3125}$
7. Find the H.C.F. of 1.08, 0.36 and 0.9.
8. The product of two numbers is 4107. If the H.C.F. of these numbers is 37, then find the greater numebr.
9. Find the L.C.M. of 148 and 185.
10. If two positive integers 'm' and 'n' can be expressed as $m = ab^2$ and $n = a^3b$; a, b being prime numbers, then find L.C.M. (m, n).

SHORT ANSWER TYPE

1. The sum of two numbers is 528 and their H.C.F. is 33, then find the number of pairs of numbers satisfying the above conditions.
2. Which of the integers (99, 101, 176, 182) has most number of divisors ?
3. Find the number of natural numbers divisible by 5 between 1 and 1000.
4. Find the number of divisors of 392.
5. There is a remainder of 3 when a number is divided by 6. What will be the remainder if the square of the same number is divided by 6 ?

LONG ANSWER TYPE

1. Let n = 640640640643, without actually computing n^2 prove that n^2 leave a remainder 1 when divided by 8.
2. Find the largest number of four digits exactly divisible by 12, 15, 18 and 27.
3. p is prime n is a positive integer and n + p = 2000. L.C.M. of n and p is 21879. Then find n and p.
4. The smallest positive integer k such that (2000) (2001) k is perfect cube.
5. If a, b, c, n are rational numbers such that

   (i) n is not a perfect cube of a rational number.

   (ii) $a + bn^{1/3} + cn^{2/3} = 0$, then prove that a = b = c = 0.

TRUE / FALSE TYPE

1. A number of the form 3q + 1 (q is any integer) is always even.
2. $0.34\overline{31}$ is irrational.
3. Division of 2 rational numbers always gives a rational numebr.
4. HCF of 143 and 85 is 7.
5. 343 and 567 are coprime.
6. The unit place digit of H.C.F. of $2^2 \times 3^2 \times 5^3 \times 7$, $2^3 \times 3^3 \times 5^2 \times 7^2$ and $3 \times 5 \times 7 \times 11$ is.

FILL IN THE BLANKS

1. If p and q are two prime numbers, then HCF.....
2. π is the.....
3. If the product of two numbers is 1080 and their HCF is 30. Then, LCM.....
4. $3 \times 5 \times 7 + 7$ is a .....
5. The sum of two prime numbers is always a .....

ANALYTICAL PROBLEM

1. The unit place digit of H.C.F. of $2^2 \times 3^2 \times 5^3 \times 7$, $2^3 \times 3^3 \times 5^2 \times 7^2$ and $3 \times 5 \times 7 \times 11$ is.
2. Three numbers are in the ratio 1 : 2 : 3 and their H.C.F. is 12. Then the square root of largest number is.
3. Find numerator in the fractional representation of $0.\overline{81}$.
4. A rectangular courtyard 3.78 metres long and 5.25 metres wide is to be paved exactly with square tiles, all of the same size. Then the largest size of the tile which could be used for the purpose is n cm. Find n.
5. The greatest number which can divide 1356, 1868 and 2764 leaving the same remainder 12 in each case, is square of _____.

NUMERICAL PROBLEMS

1. The least number that must be added to 3105 to get a number exactly divisible by 21.
2. The unit's digit in $[(264)^{102} + (264)^{103}]$ is.
3. The greatest number that will divide 103, 127 and 175, so as to leave remainder 55 in each case is.
4. Find the minimum number by which 891/3500 must be multiplied to make it a terminating decimal.
5. On dividing 12401 by a certain number, we get 76 as quotient, 13 as remainder and k as divisor. Find k.

Space for Notes :

Answer Key

EXERCISE–I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	A	C	A	D	B	B	A	D	B	B	B	B	B	C
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
B	A	A	D	C	C	A	B	A	C	D	C	B	C	A
31	32	33												
B	C	A												

EXERCISE II

VERY SHORT ANSWER TYPE

3. $\text{H.C.F.} = \dfrac{\text{Product of two numbers}}{\text{L.C.M.}}$ **4.** 980 **5.** 36 **6.** (i) Terminating, (ii) Terminating

7. 0.18 **8.** 111 **9.** 740 **10.** $a^3 \times b^2$

SHORT ANSWER TYPE

1. 4 **2.** 176 **3.** 199 **4.** 12 **5.** 3

LONG ANSWER TYPE

1. 1 **2.** 9720 **3.** p = 11, n = 1989 **4.** $k = 2^2 \times 3^2 \times 23^2 \times 29^2$

TRUE / FALSE

1. F **2.** F **3.** F **4.** F **5.** F

FILL IN THE BLANKS

1. 1 **2.** Irrational Number **3.** 36 **4.** Composite Number

5. Prime Number

ANALYTICAL PROBLEM

1. 5 **2.** 6 **3.** 9 **4.** 21 **5.** 8

NUMERICAL PROBLEMS

1. 3 **2.** 0 **3.** 24 **4.** 7 **5.** 163

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : REAL NUMBER)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

POLYNOMIALS 2

Concepts

INTRODUCTION

An algebraic expression of the form $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots\dots\dots + a_nx^n$, where 'n' is a non – negative integer and $a_0, a_1, a_2, \dots a_n$ are real numbers such that $a_n \neq 0$ is called a polynomial in 'x' of degree n.

The exponent of the variable x in every term is a whole number. For e.g., $x + 7$, $3x - 2$ are polynomials whereas $2\sqrt{x} + 3, x + \frac{1}{x}$ are not polynomials because here exponents of x are not whole numbers.

Consider the polynomial :

$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots\dots + a_nx^n$.

Here $a_0, a_1x, a_2x^2 \dots$ are terms of the polynomials and $a_0, a_1, a_2 \dots a_n$ are the respective coefficients of the term.

The coefficient of the highest degree term is called leading coefficient.

For example: consider the polynomial $f(x) = 3x^4 + x^3 - 4x + 7$.

Here, coefficient of x^3 is 1, coefficient of x^2 is 0 and there are 4 terms in f(x) and leading coefficient is 3.

1. DEGREE OF POLYNOMIAL

The highest power of x in the polynomial is called the degree of the polynomial.

For e.g., if $f(x) = 4x^3 + 3x$, then degree of f(x) is 3.

- The polynomial $f(x) = 0$ is called a zero polynomial. Degree of a zero polynomial is not defined.
- A polynomial whose degree is 0 is called a constant polynomial.

  i.e., f(x) = constant is known as a constant polynomial. For e.g., $f(x) = \frac{2}{3}$ is a constant polynomial.
- A polynomial of degree 1 is called a linear polynomial e.g., $4x + 7$.
- A polynomial of degree 2 is called a quadratic polynomial e.g., $3x^2 - 7$.
- A polynomial of degree 3 is called a cubic polynomial e.g., $2x^3 - 3x + 2$.
- A polynomial of degree 4 is called a biquadratic polynomial e.g., $3x^4 - 3x$.

Focus Point

1. An expression in a single variable having just one term is known as monomial e.g., $f(x) = 3x^2$ is a monomial.
2. An expression having two terms is known as binomial e.g., $2x + 7$, $4x^2 - 3x$, $3 - 4x^2$ are binomials.
3. An expression having 3 terms is known as a trinomial. e.g., $4x^2 + 3x + 1$, $2x^3 - 4x + 2$ are trinomials.

1.1 VALUE OF A POLYNOMIAL

The value of a polynomial f(x) at x = a is obtained by substituting x = a in f(x) and is denoted by f(a).

For e.g, if $f(x) = 2x^2 + 3$

then $f(3) = 2(3)^2 + 3 = 21$

$\therefore$ value of f(x) at x = 3 is 21.

1.2 ZEROES OR ROOTS OF A POLYNOMIAL

If value of a polynomial f(x) becomes zero at x = a, then a is called a zero or root of f(x).

or if a is the zero of the polynomial f(x) then f(a) = 0.

e.g., if $f(x) = x^2 - 3x + 2$

then $f(1) = 1 - 3 + 2 = 0$

$\therefore$ x = 1 is a root of $f(x) = x^2 - 3x + 2$.

Example 1

If x = 3 is a root of $f(x) = x^3 - 4x^2 + 7k$, then find k.

Solution :

x = 3 is a root of f(x)

$\Rightarrow f(3) = 0$

$\Rightarrow 3^3 - 4 \cdot 3^2 + 7k = 0.$

$-9 + 7k = 0$

$\therefore k = \frac{9}{7}.$

Example 2

Which of the following are not polynomials?

(a) $4x^2 + 3$ (b) $x^2 + \frac{1}{x^2}$ (c) $\sqrt{x} + \frac{7}{x}$

Solution :

$4x^2 + 3$ is a polynomial because x is raised to powers 2 is whole number but $x^2 + \frac{1}{x^2}$ and $\sqrt{x} + \frac{7}{x}$ are not polynomials, because here exponents of x are not whole numbers.

Example 3

find roots (or zeroes) of the following polynomials:

(a) $4x^2 - 1$ (b) $2x + 3$ (c) $x^3 - 1$

Solution :

(a) $4x^2 - 1 = 0 \Rightarrow x = 1/2, -1/2$ $\therefore$ the roots are x = 1/2, – 1/2

(b) $2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$ is the zero.

(c) $x^3 - 1 = 0 \Rightarrow x = 1$ is the root.

1.3 REMAINDER THEOREM

Let p(x) be any polynomial of degree greater than or equal to one and let 'a' be any real number. If p(x) is divided by the linear polynomial x – a, then the remainder is p(a). i.e., $p(x) = (x - a)\, q(x) + p(a)$

Example 4

Find the remainder when $x^4 + x^3 - 2x^2 + x + 1$ is divided by $x - 1$.

Solution :

$\Rightarrow p(1) = (1)^4 + (1)^3 - 2(1)^2 + 1 + 1 = 2$ So, the remainder is 2.

2. GEOMETRICAL MEANING OF THE ZEROES OF A POLYNOMIAL

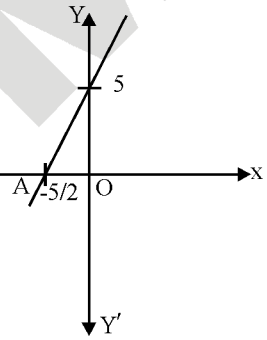
A polynomial y = p(x) has many zeroes, as the number of times it's curve come in contact (touches or cuts) with the axis of x.

2.1 LINEAR POLYNOMIAL

For a linear polynomial $ax + b$ $(a \neq 0)$, the graph of $y = ax + b$ is a straight line which intersects the x axis at exactly one point.

Let the point of intersection be $P(x_1, y_1)$. Since P lies on x–axis, its y–coordinate will be zero. So x_1 is that value of x which makes $ax + b$ zero. i.e., x_1 is a root of the polynomial $ax + b$.

From above discussion, we can say that the straight line $y = ax + b$ intersects the x – axis at the point whose x–coordinate is a zero of $ax + b$. For e.g., consider the line $y = 2x + 5$.



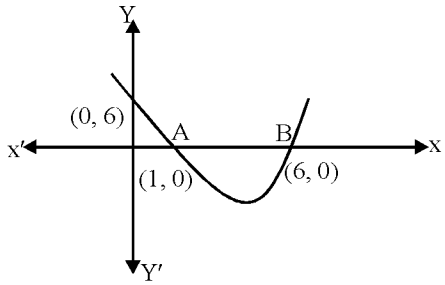
This line intersects the x–axis at $\left(-\frac{5}{2}, 0\right)$. Evidently, $x = -\frac{5}{2}$ is a zero of $2x + 5$.

2.2 QUADRATIC POLYNOMIAL

If we consider the quadratic polynomial $ax^2 + bx + c$ $(a \neq 0)$, the graph of $y = ax^2 + bx + c$ has shape of a parabola, either opening upwards $(a > 0)$ or opening downwards $(a < 0)$. The shape of the parabola may be $\cup$ or $\cap$. Just like linear polynomial, the zeroes of the quadratic polynomial $ax^2 + bx + c$ are the x–coordinates of the points where the graph of $y = ax^2 + bx + c$ intersects the x – axis.

For e.g., consider the quadratic polynomial $x^2 - 7x + 6$.

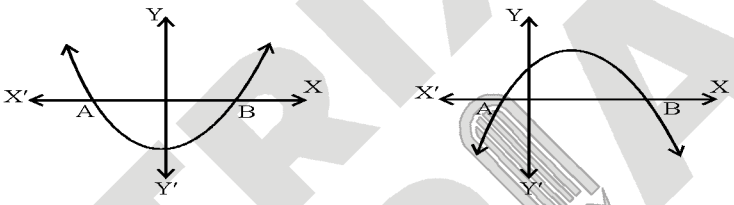
If we plot the graph of $y = x^2 - 7x + 6$, it will be as follows.



The graph intersects the x–axis at point A(1, 0) and B (6, 0) whose x–coordinates are the zeroes of $x^2 - 7x + 6$.

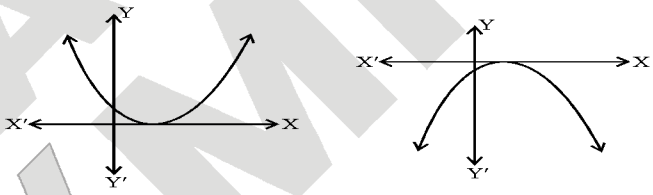
The graph of a quadratic polynomial may be one of the following:

Case 1:



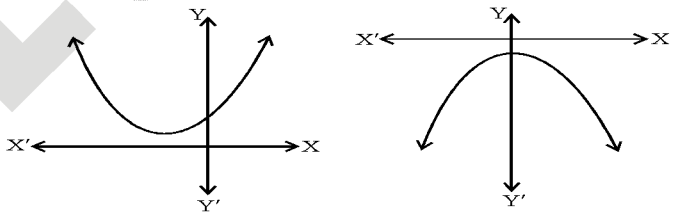
Here, the quadratic polynomial has 2 distinct roots which are x–coordinates of points A and B.

Case 2:



Here, the graph touches the x–axis at just one point i.e., 2 coincident points. Here, the quadratic polynomial has 2 equal zeroes. E.g. $y = x^2 + 2x + 3$

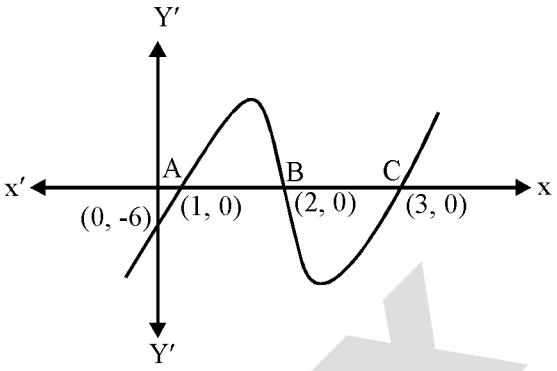
Case 3:



Here, the graph is completely above the x–axis or completely below the x–axis i.e. the quadratic polynomial has no zero. E.g. $y = x^2 + x + 1$

2.3 CUBIC POLYNOMIAL

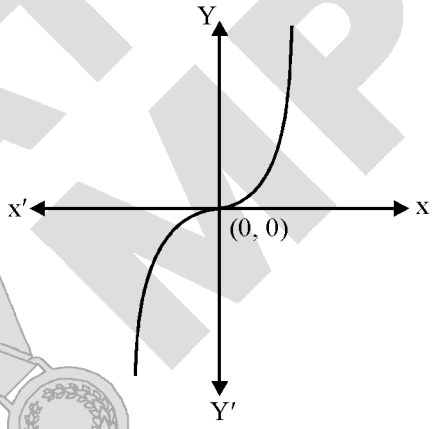
Consider the cubic polynomial $x^3 - 6x^2 + 11x - 6$. If we plot graph of $y = x^3 - 6x^2 + 11x - 6$ it will be as follows:



The given polynomial is $y = (x - 1)(x - 2)(x - 3)$ its zeroes are $x = 1$, $x = 2$ and $x = 3$.

It can be seen from the graph, the x–coordinates of the point of intersection of graph with x–axis are $x = 1$, $x = 2$ and $x = 3$.

Consider the graph of $y = x^3$



Here the graph intersects the x–axis at (0, 0). Thus $y = x^3$ has 3 coincident zeroes $x = 0$.

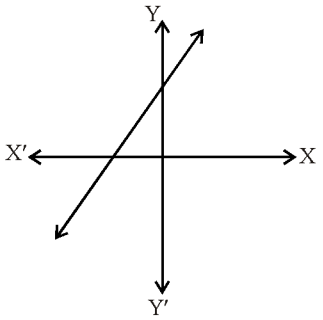
Focus Point

- In general, given a polynomial p (x) of degree n, the graph of y = p (x) intersects the x–axis at atmost n points. i.e., a polynomial p(x) of degree n has atmost n zeroes.

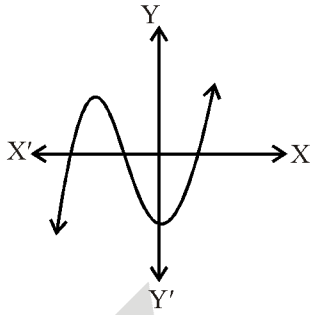
Example 5

For each of the graphs, find the number of zeroes.

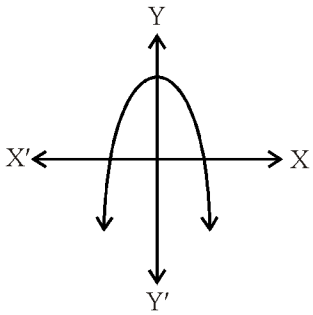
(A)



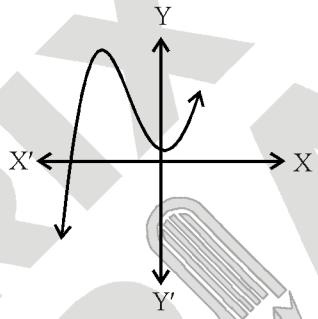
(B)



(C)



(D)



Solution :

(A) The graph intersects the x–axis at one point. So number of zeroes is 1.

(B) Similarly, here number of zeroes is 3. (C) The number of zeroes is 2

(D) Number of zeroes is 1.

3. RELATIONSHIP BETWEEN ZEROES AND COEFFICIENTS OF A POLYNOMIAL

3.1 QUADRATIC EQUATION

If α and β are the zeroes of a quadratic polynomial $p(x) = ax^2 + bx + c$ $(a \neq 0)$, then $(x - \alpha)$ and $(x - \beta)$ are its factors.

$\therefore\ ax^2 + bx + c = k(x - \alpha)(x - \beta)$, where k is a constant

$= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= kx^2 - k(\alpha + \beta)x + k\alpha\beta$

Comparing coefficients of x^2, x and constant terms on both sides.

$a = k,\ b = -k(\alpha + \beta),\ c = k\alpha\beta$

$$\Rightarrow \alpha + \beta = -\frac{b}{a}$$

$$\text{and } \alpha\beta = \frac{c}{a}$$

i.e., sum of zeroes $= -\dfrac{\text{coeff. of } x}{\text{coeff. of } x^2}$

product of zeroes $= \dfrac{\text{constant term}}{\text{coeff. of } x^2}$

Example 6

Find a quadratic polynomial, the sum and product of whose zeroes are 5 and 6 respecitvely.

Solution :

Given,

$\alpha + \beta = 5 = -\dfrac{b}{a}$

$\alpha\beta = 6 = \dfrac{c}{a}$

if a = 1, then b = –5, c = 6

$\therefore$ the quadratic polynomial is $ax^2 + bx + c$

i.e., $x^2 - 5x + 6$.

3.2 CUBIC POLYNOMIAL

If $ax^3 + bx^2 + cx + d$ is a cubic polynomial whose roots are α, β, γ, then

sum of zeroes : $\alpha + \beta + \gamma = -\dfrac{b}{a}$

sum of zeroes taking two at time : $\alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{c}{a}$

product of zeros : $\alpha\beta\gamma = -\dfrac{d}{a}$

For example consider the cubic polynomial $p(x) = x^3 - 4x^2 + x + 6$

Here $p(x) = (x - 2)(x + 1)(x - 3)$

i.e., 2, –1 and 3 are zeroes of p(x).

Now here a = 1, b = –4, c = 1, d = 6

Sum of zeroes, $\alpha + \beta + \gamma = 2 - 1 + 3 = 4$

also $-\dfrac{b}{a} = 4$ $\quad\therefore$ sum of zeroes $= -\dfrac{b}{a}$

Product of zeroes : $\alpha\beta\gamma = 2(-1)3 = -6$

Also, $-\dfrac{d}{a} = -6$ $\quad\therefore$ Product of zeroes : $-\dfrac{d}{a}$ $\quad\Rightarrow \dfrac{-6}{1} = -6$

4. DIVISION ALGORITHM FOR POLYNOMIALS

Here,we will discuss the method of long division of polynoimial by dividing one polynomial by another with the help of an example.

Consider division of $4x^3 - 3x^2 + 6x + 2$ by $2x + x^2 - 1$.

First of all, we change them into their standard form. i.e., we arrange the divisor and the dividend in decreasing powers of x.

Step 1: To obtain first term of quotient, divide the highest degree term of dividend by highest degree term of divisor i.e., $\frac{4x^3}{x^2} = 4x$. Then carry out the division process. What remains is $-11x^2 + 10x + 2$.

Step 2: For second term of quotient, divide the highest degree term of new dividend by highest degree term of divisor i.e, $-\frac{11x^2}{x^2} = -11$. Again carry out the division. What remains is $32x - 9$. Now degree of $32x - 9$ is less than the degree of divisor. Therefore, we cannot continue with the division any longer.

$$
\begin{array}{r|l}
 & 4x - 11 \\
\hline
x^2 + 2x - 1 & 4x^3 - 3x^2 + 6x + 2 \\
 & \underline{-4x^3 \pm 8x^2 \mp 4x} \\
 & -11x^2 + 10x + 2 \\
 & \underline{\mp 11x^2 \mp 22x \pm 11} \\
 & 32x - 9
\end{array}
$$

Here, dividend = $4x^3 - 3x^2 + 6x + 2$, the divisor is $x^2 + 2x - 1$, the quotient is $4x - 11$, remainder is $32x - 9$ and We know that dividend = divisor × quotient + remainder. (Euclid's Division Lemma)

Here also, the above relation holds true.

Division Algorithm : If p(x) and q(x) are two polynomials with $q(x) \neq 0$, then we can find 2 polynomials r(x) and s(x) such that $p(x) = s(x) \times q(x) + r(x)$.

where r(x) will be 0 or degree of r(x) < degree of q(x). This result is known as division algorithm for polynomials.

Example 7

Divide $4x^4 - 3x + 2x^2 + 6$ by $x - 3$.

Solution :

First, change given polynomials in standard form,

i.e., $4x^4 + 2x^2 - 3x + 6$ and $x - 3$.

Then, division can be done as follws :

$$
\begin{array}{r|l}
 & 4x^3 + 12x^2 + 38x + 111 \\
\hline
x - 3 & 4x^4 + 2x^2 - 3x + 6 \\
 & \underline{\underset{-}{4x^4} \qquad\qquad \underset{+}{-}12x^3} \\
 & 12x^3 + 2x^2 - 3x + 6 \\
 & \underline{\underset{-}{12x^3} \underset{+}{-} 36x^2} \\
 & 38x^2 - 3x \quad 6 \\
 & \underline{\underset{-}{38x^2} \underset{+}{-} 114x} \\
 & 111x + 6 \\
 & \underline{\underset{-}{111x} \underset{+}{-} 333} \\
 & 339
\end{array}
$$

Here, quotient = $4x^3 + 12x^2 + 38x + 111$ and remainder = 339.

5. HCF AND LCM OF POLYNOMIALS

If a polynomial f(x) is a product of two polynomials g(x) and h(x) i.e.; f(x) = g(x).h(x) then g(x) and h(x) are called factors of f(x).

Example 8

If $f(x) = x^2 - 5x + 6$.

i.e. $f(x) = x^2 - 5x + 6 = (x - 2)(x - 3)$

then $(x - 2)$ and $(x - 3)$ are factors of $x^2 - 5x + 6$.

Focus Point

- If g(x) is a factor of f(x) then '–g(x)' is also a factor of f(x). Generally we take 'g(x)' or '–g(x)' as a factor in which the highest degree term has positive coefficient.

5.1 HCF OF POLYNOMIALS

The highest common factor (HCF) of two polynomials f(x) and g(x) is that common factor which has highest degree among all common factors and in which the coefficient of highest degree term is positive.

Working Rule : To find the HCF of two or more given polynomials.

Step 1. Express each polynomial as a product of powers of irreducible factors (simple factors). Numerical factors, if any, are expressed as product of powers of prime numbers.

Step 2. If there is no common factor, then the HCF is 1. If there are common simple factors find the smallest exponents of these simple factors in the factorised form of the polynomials.

Step 3. Raise the common simple factors to the smallest exponents found in step 2 and multiply to get the HCF.

Example 9

Find the HCF of the polynomials.

$150(6x^2 + x - 1)(x - 3)^3$ and $84(x - 3)^2(8x^2 + 14x + 5)$.

Solution :

Let $f(x) = 150(6x^2 + x - 1)(x - 3)^3$

and $g(x) = 84(x - 3)^2 (8x^2 + 14x + 5)$

Now : $f(x) = 150(6x^2 + x - 1)(x - 3)^3$

$= 2 \times 3 \times 5^2 (2x + 1)(3x - 1)(x - 3)^3$

$g(x) = 84(x - 3)^2 (8x^2 + 14x + 5)$

$= 2^2 \times 3 \times 7(x - 3)^2 (2x + 1)(4x + 5)$

Common simple factor	Least exponent
2	1
3	1
$2x + 1$	1
$x - 3$	2

Hence, required HCF $= 2^1.3^1(2x + 1)^1(x - 3)^2 = 6(2x + 1)(x - 3)^2$

5.2 LCM OF POLYNOMIALS

The least common multiple (LCM) of two or more polynomials is the polynomial of the lowest degree, having smallest numerical coefficient which is exactly divisible by the given polynomials and whose coefficient of highest degree term has the same sign as the sign of the coefficient of highest degree term in their product.

Working rule : To find out LCM of two or more polynomials, we may use the following three steps

Step 1. Express each polynomial as a product of powers of irreducible factors. Express numerical factors, if any, as product of powers of prime numbers.

Step 2. Consider all the irreducible factors occurring in the given polynomials each one once only. Find the greatest exponent of each of these simple factors in the factorised form of the given polynomials.

Step 3. Raise each irreducible factor to the greatest exponent found in step 2, and multiply to get the LCM.

Example 10

Find the LCM of the polynomials

$90(x^2 - 5x + 6)(2x + 1)^2$ and $140(x - 3)^3 (2x^2 + 15x + 7)$.

Solution :

Let $f(x) = 90(x^2 - 5x + 6)(2x + 1)^2$

and $g(x) = 140(x - 3)^3 (2x^2 + 15x + 7)$

Then $f(x) = 2 \times 3^2 \times 5 (x - 2)(x - 3)(2x + 1)^2$

and $g(x) = 2^2 \times 5 \times 7(x - 3)^3 (2x + 1)(x + 7)$

Irreducible factor	Greatest exponent
2	2
3	2
5	1
7	1
x – 2	1
x – 3	3
2x + 1	2
x + 7	1

$LCM = 2^2.3^2.5^1.7^1.(x - 2)^1. (x - 3)^3 (2x + 1)^2 .(x + 7)^1$

$LCM = 1260(x - 2)(x - 3)^3 (2x + 1)^2(x + 7)$

5.3 RELATION BETWEEN LCM AND HCF

◊ L.C.M. {of f(x) and g(x)} × H.C.F {of f(x) and g(x)} = – f(x).g(x)

◊ Thus one should use this result with caution. Four quantities viz, LCM, HCF, f(x) and g(x) are involved here.

◊ If f(x), g(x) and one of LCM., HCF. are given, the other can be found without ambiguity because L.C.M. and H.C.F. are unique, except for a factor of –1.

◊ If H.C.F., L.C.M. and one of f(x) and g(x), say f(x) are given, then g(x) can not be determined uniquely. There would be two polynomials g(x) and –g(x) which with f(x), produce the given H.C.F and L.C.M.

◊ Let us denote the two polynomials by f(x) and g(x). Let us denote the H.C.F. by h(x) and the L.C.M. by l(x). Then we have : $h(x) \times l(x) = \pm f(x) \times g(x)$

Example 11

The H.C.F. of the polynomials $p(x) = (x-3)(x^2+x-2)$ and $q(x) = x^2 - 5x + 6$ is $x - 3$. Find the L.C.M.

Solution :

$p(x) = (x-3)(x^2+x-2) = (x-3)(x-1)(x+2)$

$q(x) = (x^2-5x+6) = (x-3)(x+2)$

H.C.F. $= (x-3)$

$$\text{L.C.M} = \frac{p(x)\times q(x)}{\text{H.C.F}} = \frac{(x-3)(x-1)(x+2)(x-3)(x-2)}{x-3}$$

$= (x-1)(x+2)(x-3)(x-2)$.

Example 12

The L.C.M and H.C.F. of two polynomials, p(x) and q(x) are $2(x^4-1)$ and $(x+1)(x^2+1)$ respectively. If $p(x) = x^3+x^2+x+1$, then find q(x).

Solution :

$p(x) = x^3+x^2+x+1 = (x+1)(x^2+1)$

$p(x) \times q(x) = \pm$ L.C.M. $\times$ H.C.F

$$q(x) = \pm\frac{\text{L.C.M}\times\text{H.C.F}}{p(x)}$$

$$= \pm\frac{2(x^4-1)\times(x+1)(x^2+1)}{(x+1)(x^2+1)}$$

$$= \pm 2(x^4-1) = \pm 2x^4 - 2$$

SOLVED EXAMPLES

SE. 1

Divide $x^4 + 3x^3 - 7x^2 + 6x + 2$ by $x^2 - 1$ and find the remainder.

Ans.

$$
\begin{array}{r|l}
 & x^2 + 3x - 6 \\
\hline
x^2 - 1 & x^4 + 3x^3 - 7x^2 + 6x + 2 \\
 & \underline{x^4 \qquad - x^2} \\
 & \quad - \qquad\quad + \\
 & \quad 3x^3 - 6x^2 + 6x \\
 & \quad \underline{3x^3 \qquad - 3x} \\
 & \qquad - 6x^2 + 9x + 2 \\
 & \qquad \underline{- 6x^2 \qquad + 6} \\
 & \qquad + \qquad\quad - \\
 & \qquad\qquad 9x - 4
\end{array}
$$

$\therefore$ the remainder is $9x - 4$.

SE. 2

Find all the zeroes of $3x^4 - 7x^3 - 7x^2 + 21x - 6$ given that $\sqrt{3}$ and $-\sqrt{3}$ are two of its zeroes.

Ans. Given that $\sqrt{3}$ and $-\sqrt{3}$ are zeroes.

$\therefore \left(x - \sqrt{3}\right)$ and $\left(x + \sqrt{3}\right)$ are factors of given polynomial i.e.,

$\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right) = x^2 - 3$ is a factor. We divide the given polynomial by $x^2 - 3$.

$$
\begin{array}{r|l}
 & 3x^2 - 7x + 2 \\
\hline
x^2 - 3 & 3x^4 - 7x^3 - 7x^2 + 21x - 6 \\
 & \underline{3x^4 \qquad - 9x^2} \\
 & - \qquad\quad + \\
 & - 7x^3 + 2x^2 + 21x \\
 & \underline{- 7x^3 \qquad + 21x} \\
 & \qquad 2x^2 \qquad - 6 \\
 & \qquad \underline{2x^2 \qquad - 6} \\
 & \qquad\qquad 0
\end{array}
$$

$\therefore 3x^4 - 7x^3 - 7x^2 + 21x - 6 = (x^2 - 3)(3x^2 - 7x + 2)$
$= (x^2 - 3)(3x - 1)(x - 2)$

So its zeroes are given by $\sqrt{3}, -\sqrt{3}, \frac{1}{3}, 2$

SE. 3

$f(x) = 2x^3 + 5x^2 + 6x + 7$, find $f(-1)$.

Ans. $x = -1$ to put in given $f(x)$ for $f(-1)$

$f(-1) = 2(-1)^3 + 5(-1)^2 + 6(-1) + 7$
$= -2 + 5 - 6 + 7 = 4$

SE. 4

Find roots of polynomial $x^2 - 16$

Ans. $x^2 - 16 = 0 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$
$\Rightarrow x = -4, +4$ are roots

SE. 5

$x\sqrt{x} + 5x$ is a polynomial or not ? Give reason.

Ans. No, exponents of x in first term is $\frac{3}{2}$ which is not whole number.

SE. 6

Consider cubic polynomial $x^3 + 2x^2 + 3x + 4$. What is sum of all roots.

Ans. Sum $\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{2}{1} = -2$

SE. 7

If $x = 2$ is root of $f(x) = x^3 - x + k$, then find k.

Ans. $x = 2$ is root of $f(x) \Rightarrow f(2) = 0 \Rightarrow (2)^3 - 2 + k = 0 \Rightarrow 8 - 2 + k = 0 \therefore k = -6$

SE. 8

Find the quadratic polynomial if sum and product of zeroes are $\frac{3}{2}$ and $\frac{5}{2}$ respectively.

Ans. Given $\alpha + \beta = \frac{3}{2} = -\frac{b}{a}$; $\alpha.\beta = \frac{5}{2} = \frac{c}{a}$
$\Rightarrow a = 2, b = -3, c = 5$
Quadratic polynomial is $ax^2 + bx + c$
i.e., $2x^2 - 3x + 5$.

SE. 9

Check whether first polynomial is factor of second polynomial or not $x - 3$, $x^2 - 9$.

Ans. Second polynomial can written as
$x^2 - 9 = (x + 3)(x - 3)$ which contains $x - 3$
hence $x - 3$ is factor of $x^2 - 9$.

EXERCISE – I

ONLY ONE CORRECT TYPE

1. Which of the following is not a polynomial ?

(A) $x^3 + 1$ (B) $x + \frac{1}{x}$

(C) $x^2 - x$ (D) None of these

2. Which of the following polynomial has degree 2

(A) $\frac{x^2+1}{x^2}$ (B) $3x^2 + 5$

(C) $4x^3 - 3x^2 + 7$ (D) $8x^3 - 2$

3. Sum of zeroes of polynomial $P(x) = 4x^2 - 1$ is

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$

(C) 0 (D) 4

4. The quadratic polynomial whose zeroes are 0 and $\sqrt{2}$ is

(A) $x^2 - \sqrt{2}x + 1$ (B) $2x^2 - \sqrt{2}x + 3$

(C) $x^2 - \sqrt{2}x$ (D) None of these

5. When $p(x) = x^3 + ax^2 + 2x + a$ is divided by $(x + a)$; the remainder is

(A) 0 (B) a

(C) –a (D) 2a

6. In the graph of a polynomial intersects the x-axis in 3 points, then its degree cannot be

(A) 2 (B) 3

(C) 4 (D) 5

7. If $4x^4 - 3x^3 - 3x^2 + x - 7$ is divided by $1 - 2x$ then remainder will be

(A) $\frac{57}{8}$ (B) $-\frac{59}{8}$

(C) $\frac{55}{8}$ (D) $-\frac{55}{8}$

8. The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ when divided by $(x - 4)$ leaves remainders R_1 & R_2 respectively then value of 'a' if $2R_1 - R_2 = 0$.

(A) $-\frac{18}{127}$ (B) $\frac{18}{127}$

(C) $\frac{17}{127}$ (D) $-\frac{17}{127}$

9. A quadratic polynomial is exactly divisible by $(x + 1)$ & $(x + 2)$ and leaves the remainder 4 after division by $(x + 3)$ then that polynomial is

(A) $x^2 + 6x + 4$ (B) $2x^2 + 6x + 4$

(C) $2x^2 + 6x - 4$ (D) $x^2 + 6x - 4$

10. The values of a & b so that the polynomial $x^3 - ax^2 - 13x + b$ is divisible by $(x - 1)$ & $(x + 3)$ are

(A) a = 15, b = 3 (B) a = 3, b = 15

(C) a = – 3, b = 15 (D) a = 3, b = – 15

11. Graph of quadratic equation is always a –

(A) Straight line (B) Circle

(C) Parabola (D) Hyperbola

12. If the sign of 'a' is positive in a quadratic polynomial $ax^2 + bx + c$ then its graph should be

(A) Parabola open upwards

(B) Parabola open downwards

(C) Parabola open leftwards

(D) Can't be determined

13. The graph of polynomial $p(x) = x^3 - x^2 + x$ is always passing through the point –

(A) (0, 0) (B) (3, 2)

(C) (1, – 2) (D) All of these

14. How many time, graph of the polynomial $f(x) = x^3 - 1$ will intersect X – axis –

(A) 0 (B) 1

(C) 2 (D) 4

15. Which of the following curve touches X – axis –

(A) $x^2 - 2x + 4$ (B) $3x^2 - 6x + 1$

(C) $4x^2 - 16x + 9$ (D) $25x^2 - 20x + 4$

16. If $x^2 + xy + x = 12$ and $y^2 + xy + y = 18$, then the vlaue of x + y is.....

(A) 5, – 6 (B) 3, 4

(C) 5, 3 (D) 6, – 3

17. How many zeros can a polynomial of degree n have

(A) n + 1 (B) n – 1

(C) n (D) n^2

18. If $(x + 1)$ is a factor of $x^2 - 3ax + 3a - 7$. Then the value of a is :
(A) 1 (B) – 1
(C) 0 (D) 2

19. If one of the zeroes of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is (-3), then k is equal to :
(A) $\frac{4}{3}$ (B) $-\frac{4}{3}$
(C) $\frac{2}{3}$ (D) $-\frac{2}{3}$

20. The quadratic polynomial whose sum of zeroes is 3 and product of zeroes is – 2 is :
(A) $x^2 + 3x - 2$ (B) $x^2 - 2x + 3$
(C) $x^2 - 3x + 2$ (D) $x^2 - 3x - 2$

21. If one of the zeroes of polynomial $f(x) = 9x^2 + 13x + 6a$ is reciprocal of the other, then a is equal to:
(A) $\frac{1}{9}$ (B) $\frac{2}{3}$
(C) $\frac{3}{2}$ (D) $\frac{1}{6}$

22. Graph of a linear polynomial is :
(A) straight line (B) circle
(C) ellipse (D) parabola

23. If α and β are the zeroes of the polynomial $2 - 3x - x^2$ then $\alpha + \beta =$
(A) 2 (B) 3
(C) 1 (D) none

24. A quadratic polynomial whose sum of the zeroes is 3 and one of the zero is 0, is :
(A) $x^2 + 2x$
(B) $x^2 + 3x$
(C) $x^2 - 3x + 5$
(D) $x(x - 3)$

25. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and – 3, then
(A) a = 0, b = – 6
(B) a = 2, b = – 6
(C) a = 5, b = – 1
(D) a = – 7, b = – 1

PARAGRAPH TYPE

PASSAGE # I

Sum of zeroes = $\alpha + \beta = -8$ and product of zeroes = $\alpha\beta = 6$.

26. A polynomial whose zeroes are, $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is :
(A) $6x^2 + 8x + 1$ (B) $6x^2 - 8x - 1$
(C) $6x^2 - 4x + 6$ (D) $6x^2 - 8x + 1$

27. A polynomial whose zeroes are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$, is :
(A) $x^2 - 13x + 1$ (B) $36x^2 - 52x + 1$
(C) $x^2 + 13x + 9$ (D) $36x^2 + 52x - 1$

28. A polynomial whose zeroes are $(\alpha - \beta)$ and $(\alpha + \beta)$, is :
(A) $x^2 - (8 \pm 2\sqrt{10})x + 16\sqrt{10}$
(B) $x^2 - (8 \pm 2\sqrt{10})x + 16\sqrt{10}$
(C) $x^2 + (8 \pm 2\sqrt{10})x + 16\sqrt{10}$
(D) $x^2 + (8 \mp 2\sqrt{10})x - 16\sqrt{10}$

PASSAGE # II

If α and β be the zeroes of the polynomial $ax^2 + bx + c$, then the value of

29. $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}}$ is :
(A) b (B) $\frac{-b}{\sqrt{ac}}$
(C) $-\frac{\sqrt{b}}{ac}$ (D) $\frac{1}{ac}$

30. $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ is :
(A) $\frac{b^2 - 2ac}{c^2}$ (B) $\frac{b^2 + 2ac}{c^2}$
(C) $b^2 - ac$ (D) $\frac{b^2 + 4ac}{c^2}$

31. $\frac{1}{\alpha}+\frac{1}{\beta}$ is :

(A) $\frac{-b}{ac}$ (B) $b = ac$

(C) $\frac{-b}{c}$ (D) $-\sqrt{\frac{b}{ac}}$

MATCH THE COLUMN TYPE

32. Match the following :

Column – I	Column – II
(P) Number of real zeroes of $x^2 + 3$	(i) 1
(Q) One of the zeroes of $x^3 + 13x^2 + 32x + 20$ is	(ii) – 10
(R) Number of points in which $x + 3$ intersects the x-axis	(iii) 0
(S) Sum of zeroes P(x) $= 4x^3 - 8x^2 + 2x + 1$	(iv) 2

(A) (P) → (ii), (Q) → (iv), (R) → (ii), (S) → (iii)
(B) (P) → (iii), (Q) → (ii), (R) → (i), (S) → (iv)
(C) (P) → (iii), (Q) → (i), (R) → (iv), (S) → (ii)
(D) (P) → (iv), (Q) → (iii), (R) → (i), (S) → (ii)

33. If α and β are the zeroes of the polynomial $2x^2 - 4x + 5$, then match the value of List – I with that of List – II.

List – I	List – II
(P) $\frac{1}{\alpha}+\frac{1}{\beta}$	(i) – 6
(Q) $(\alpha - \beta)^2$	(ii) $\frac{-4}{25}$
(R) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$	(iii) $\frac{-2}{5}$
(S) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$	(iv) $\frac{4}{5}$

(A) (P) → (iv), (Q) → (i), (R) → (ii), (S) → (iii)
(B) (P) → (iv), (Q) → (ii), (R) → (i), (S) → (iii)
(C) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)
(D) (P) → (i), (Q) → (iv), (R) → (ii), (S) → (iii)

EXERCISE – II

VERY SHORT ANSWER TYPE

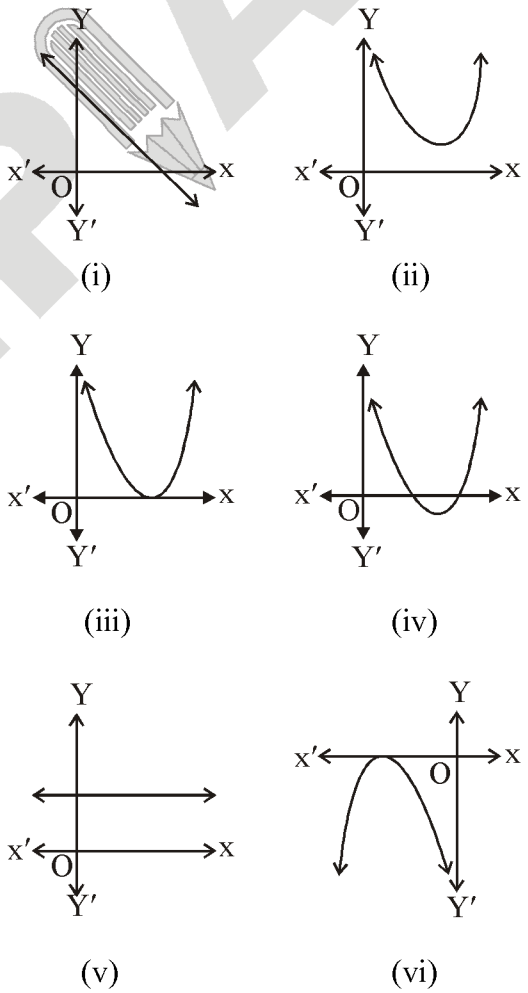
1. Draw the graph of the polynomial $f(x) = 2x - 5$. Also, find the coordinates of the point where it crosses X-axis.
2. Draw the graph of the polynomial $f(x) = x^2 - 2x - 8$.
3. If $x = \frac{4}{3}$ is a root of the polynomial $f(x) = 6x^3 - 11x^2 + kx - 20$, then find the value of k.
4. If $x = 2$ and $x = 0$ are roots of the polynomials $f(x) = 2x^3 - 5x^2 + ax + b$, then find the values of a and b.
5. What must be added to the polynomial $f(x) = x^4 + 2x^3 - 2x^2 + x - 1$, so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$.
6. What should be subtracted from the polynomial $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$, so that the resulting polynomial is exactly divisible by $x^2 - 4x + 3$.
7. If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c$. Find the value of
   (i) $\alpha^2 - \beta^2$ (ii) $\alpha^3 + \beta^3$
8. If the sum of the squares of zeroes of the polynomial $6x^2 + x + k$ is $\frac{25}{36}$, find the value of k.
9. If one zero of the quadratic polynomial $2x^2 - (3k + 1)x - 9$ is negative of the other, find the value of k.
10. If α, β are the zeroes of the polynomial $x^2 + px + q$, prove that $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} + 2 - \frac{4p^2}{q}$.

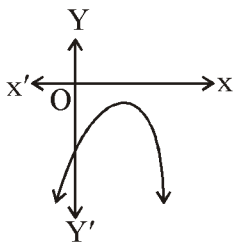
SHORT ANSWER TYPE

1. Find the zeroes of following polynomials
   (A) $\frac{1}{2}x^2 - 3x + 4$ (B) $3x^3 - 5x^2 - 11x - 3$
2. Polynomial $p(x) = -x^3 + 3x^2 - 3x + 5$ when divided by some other polynomial q(x) gives quotient $x - 2$ and remainder 3. Find q(x).
3. If $2x^3 + ax^2 + bx - 6$ has $(x - 1)$ as a factor and leaves a remainder 2 when divided by $(x - 2)$, find a and b.
4. For what value of k, is –2 a zero of the polynomial $3x^2 + 4x + 2k$?
5. Find the remainder when $9x^3 - 3x^2 + x - 5$ is divided by $x - \frac{2}{3}$

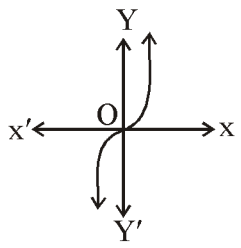
LONG ANSWER TYPE

1. Which of the following correspond to the graph to a linear or a quadratic polynomial and find the number of real zeroes of polynomial.

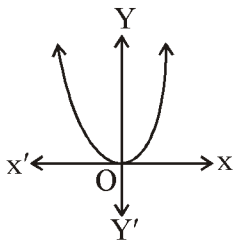




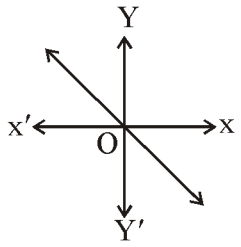
(vii)



(viii)



(ix)



(x)

2. If p and q are zeroes of the quadratic polynomial $2x^2 + 2(m + n)x + m^2 + n^2$, form the quadratic polynomial whose zeroes are $(p + q)^2$ and $(p - q)^2$.

3. If α and β are the zeroes of the polynomial $x^2 + 4x + 3$, form the polynomial whose zeroes are $1+\frac{\beta}{\alpha}$ and $1+\frac{\alpha}{\beta}$.

4. Obtain all the zeroes of $x^4 + 2x^3 - 7x^2 - 8x + 12$, if two of its zeroes are 2 and – 2.

5. Find all zeros of polynomial.
$f(x) = 2x^4 + x^3 - 4x^2 - 19x - 6$ If two of its zeros are –2 and –1.

TRUE / FALSE TYPE

1. Number of zeroes of $3x^2 + x + 1$ is 2.
2. Product of zeroes of $x^3 - 4x^2 + 6x$ is 0.
3. $-\frac{1}{\sqrt{5}}$ and $-\frac{3}{\sqrt{5}}$ are the zeroes of $5\sqrt{5}x^2 + 20x + 3\sqrt{5}$.
4. $x^2 + 4x + 7$ divides $x^3 + 3x^2 + 3x + 7$.
5. Degree of $x^3 - 4x^5 + 7$ is 3.

FILL IN THE BLANKS

1. The LCM of $14x^2y^3$ & $8x^3y^2$ is _____ .
2. The LCM of $2(x-3)^3$ & $6(x - 2)(x - 3)$ is ____ .
3. The HCF of $24a^2b$ & $40ab^2$ is ____ .
4. The HCF of $9a^2-25b^2$ & $15a^2 - 25ab$ is ____ .
5. The additive inverse of $\frac{y^2+2}{y^2-1}$ is ____ .

NUMERICAL PROBLEMS

1. Zeroes of a quadratic polynomial are in the ratio 2 : 3 and their sum is 15. The product of zeroes of this polynomial is.
2. The sum and product of zeroes of $p(x) = 63x^2 - 7x - 9$ are S and P respectively. Find the value of 27S + 14P.
3. If the polynomial $6x^4 + 8x^3 + 17x^2 + 25x - 9$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be a + bx, find $(a + b)^2$.
4. A polynomial of degree 7 is divided by a polynomial of degree 4. Degree of the quotient is
5. If $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$, then find 5a + 2b.

Answer Key

EXERCISE–I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
B	B	C	C	C	A	B	B	B	B	C	A	A	B	D
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	C	A	A	D	C	A	D	D	A	A	B	D	B	A
31	32	33												
C	B	A												

EXERCISE II

VERY SHORT ANSWER TYPE

1. $\left(\frac{5}{2}, 0\right)$ **3.** k = 19 **4.** a = 2, b = 0 **5.** x – 2 **6.** 2x – 3 **7.** (i) $-\frac{b\sqrt{b^2-4ac}}{a^2}$,

(ii) $\frac{-b^3+3abc}{a^3}$ **8.** – 2 **9.** $\frac{-1}{3}$

SHORT ANSWER TYPE

1. $a \to 4, 2$ $b \to 3, -1, -\frac{1}{3}$ **2.** $-x^2 + x - 1$ **3.** a = –8, b =12 **4.** –2 **5.** –3

LONG ANSWER TYPE

1.(i) Linear polynomial, one **(ii)** Quadratic polynomial, zero **(iii)** Quadratic polynomial, one

(iv) Quadratic polynomial, two **(v)** Linear polynomial, zero **(vi)** Quadratic polynomial, one

(vii) Quadratic polynomial, zero **(viii)** Cubic polynomial, one **(xi)** Quadratic polynomial, one

(x) Linear, one **2.** $x^2 - 4mnx - (m^2 - n^2)^2$ **3.** $3x^2 - 16x + 16$ **4.** – 3, 1 **5.** $-\frac{1}{2}$, 3, -2, -1

TRUE / FALSE

1. T 2. T 3. T 4. F

FILL IN THE BLANKS

1. $56x^3y^3$ 2. $6(x-2)(x-3)^3$ 3. 8ab 4. 3a – 5b 5. $\frac{2+y^2}{1-y^2}$

NUMERICAL PROBLEMS

1. 54 **2.** 1 **3.** 81 **4.** 3 **5.** 19

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : POLYNOMIALS)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

3

Concepts

INTRODUCTION

1. LINEAR EQUATION

A linear equation is one in which all non constant terms have degree1. For example : $3x = 7$

2. LINEAR EQUATION IN TWO VARIABLES

Linear equation in two variables is a equation containing two variables with degree one.

For example : $2x + 3y = 7$

1. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Pair of linear equations in two variables is a collection of two linear equations involving the same set of variables.

For example : $3x + 2y = 1$ and $2x - 2y = 2$ are pair of linear equations.

1.1 GEOMETRICAL INTERPRETATION

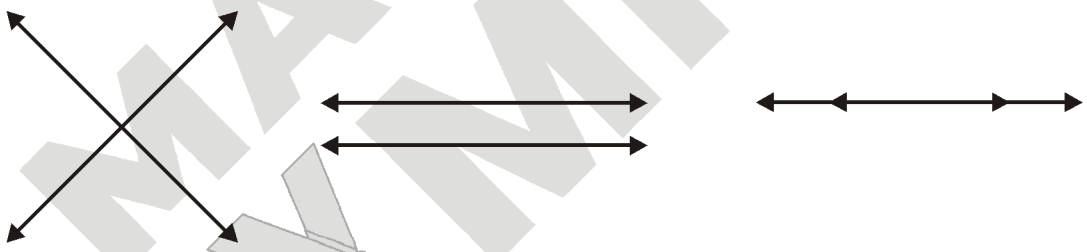
Graphically a linear equation represents a straight line. Therefore, pair of linear equations represents two straight lines. There are three distinct possibilities as given below :

(i) The two lines intersect at exactly one point.

(ii) The two lines will not intersect however far they be extended, i.e., the two lines are parallel.

(iii) The two lines are coincident, i.e., both lines overlap each other.

Graphically these can represented as



Lines intersecting at one point | Parallel lines | Coincident lines

(i) Geometrical interpretation of intersecting lines

For example : $x + y - 3 = 0$, $2x + 3y - 7 = 0$

Let $x + y - 3 = 0$ ..... (1)

$2x + 3y - 7 = 0$ ..... (2)

From (1) $x = 3 - y$

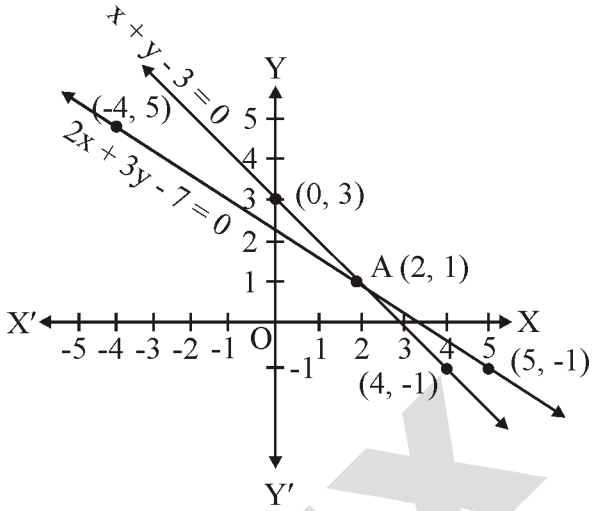
Table of solutions for equation (1) :

x	0	4
y	3	-1

From (2), $y = \dfrac{7-2x}{3}$

Table of solutions for equation (2) :

x	-4	5
y	5	-1



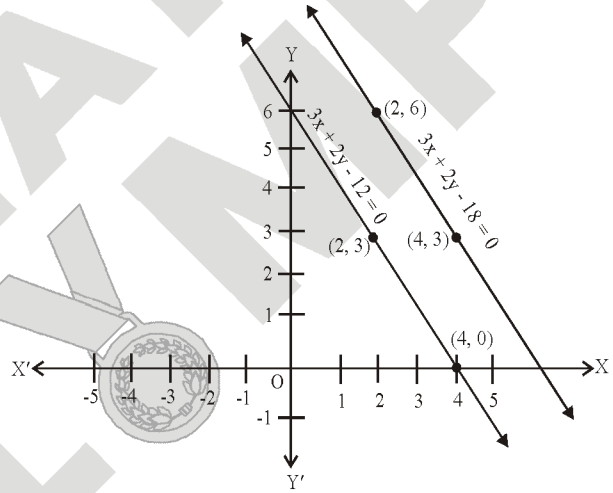
The two straight lines corresponding to the equations (1) and (2) intersect each other at one and only one point A(2, 1).

(ii) Geometrical interpretation of parallel lines

For example : $3x + 2y = 12$, $3x + 2y = 18$

$3x + 2y = 12$ i.e., $3x + 2y - 12 = 0$ .....(1)

and $3x + 2y = 18$ i.e, $3x + 2y - 18 = 0$ .....(2)



From (1), $y = \dfrac{12-3x}{2}$;

Table of solutions for equation (1) :

x	2	4
y	3	0

From (2), $y = \dfrac{18-3x}{2}$;

Table of solutions for equation (2) :

x	2	4
y	6	3

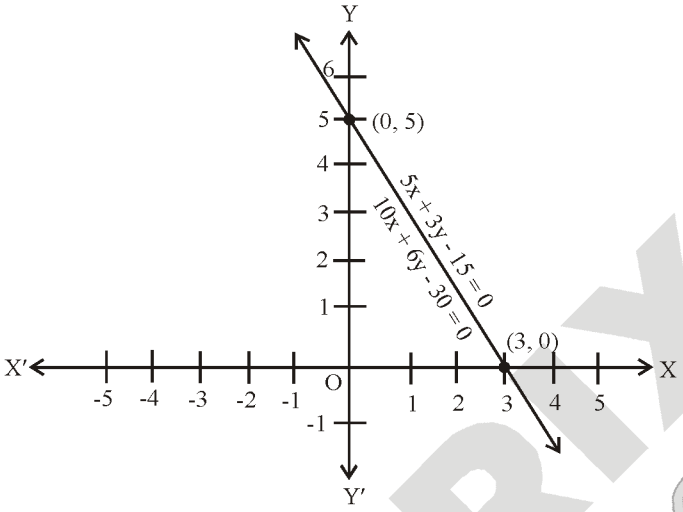
Here, the two lines do not intersect, i.e., the two lines are parallel to each other.

(iii) Geometrical interpretation of coincident lines

For example : $5x + 3y - 15 = 0$, $10x + 6y - 30 = 0$

$5x + 3y - 15 = 0$ .....(1)

and $10x + 6y - 30 = 0$ .....(2)



If we divide equation (2) by 2, then it becomes the equation (1). Thus, equations (1) and (2) will be identical and for both the equations, we have $y = \frac{15 - 5x}{3}$;

Table of solutions for equations (1) and (2) :

x	0	3
y	5	0

Here, the graph of the two equations are straight lines which coincide in a single line.

(Here, one equation is a constant multiple of the other equation).

1.2 TYPES OF PAIR OF LINEAR EQUATIONS

Inconsistent Pair of Linear Equations : A pair of linear equations which has no solution, is called an inconsis tent pair of linear equations.

Consistent Pair of Linear Equations : A pair of linear equations in two variables, which has a solution is called a consistent pair of linear equations.

Dependent Pair of Linear Equations : A pair of linear equations which are equivalent has infinitely many distinct common solutions, such a pair is called a dependent pair of linear equations in two variables. Dependent pair of linear equations is always consistent.

2. SOLUTION OF A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

For any equation $Ax + By + C = 0$, solution is an ordered pair that satisfies both the equations. There are two methods to solve the pair of linear equations in two variables.

(i) Graphical Method

(ii) Algebraic Method

2.1 GRAPHICAL METHOD FOR SOLUTION OF PAIR OF LINEAR EQUATIONS

Pair of linear equations when represented graphically is shown by two straight lines. The following cases arises when the pair of linear equations are represented graphically.

(i) The lines may intersect at a single point. Here, the pair of equations has unique solution (Intersection point) (i.e., consistent pair of equations).

(ii) The lines may be parallel. In other words, the equations have no solution (i.e., inconsistent pair of equations).

(iii) The lines may be coincident. In this case, the equations formed has infinitely many solutions (i.e., consistent pair of equations).

Let us see the examples discussed previously, geometrically.

(i) $x + y - 3 = 0$ and $2x + 3y - 7 = 0$ (The lines intersect)

(ii) $3x + 2y - 12 = 0$ and $3x + 2y - 18 = 0$ (The lines are parallel)

(iii) $5x + 3y - 15 = 0$ and $10x + 6y - 30 = 0$ (The lines coincide)

Geometrical interpretation of graphical method

(i) **Intersecting lines :** Lines l_1 and l_2 intersects at a point (a, b). So the system has unique solution which corresponds to point of intersection. (Figure (i))

(ii) **Parallel lines :** Lines l_1 and l_2 are parallel. As there is no point of intersection, we say the system has no solution. (Figure (ii))

(iii) **Coincident lines :** Lines l_1 and l_2 are coincident. As every point of l_1 lies on l_2 and l_2 has infinite points, so infinite solutions exists. Figure (iii)

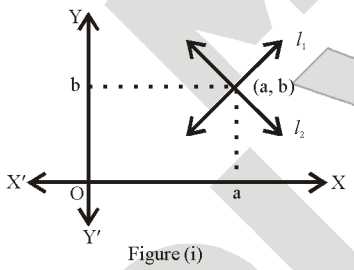


Figure (i)

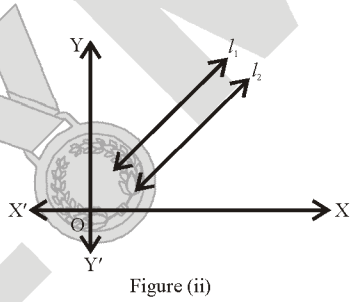


Figure (ii)

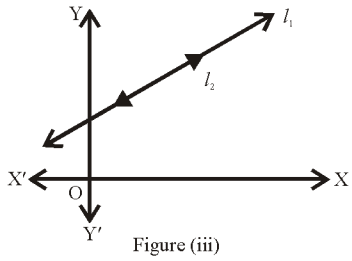


Figure (iii)

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, finding nature of solution of pair of linear equations

Two linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

(i) intersecting, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (ii) parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (iii) coincident, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

In fact, the converse is also true for any pair of lines.

Example 1

The path of highway number 1 is given by the equation $x + y = 7$ and the highway number 2 is given by the equation $5x + 2y = 20$. Represent these equations geometrically.

Solution :

We have

$$x + y = 7$$

$$\Rightarrow \quad y = 7 - x$$

In tabular form :

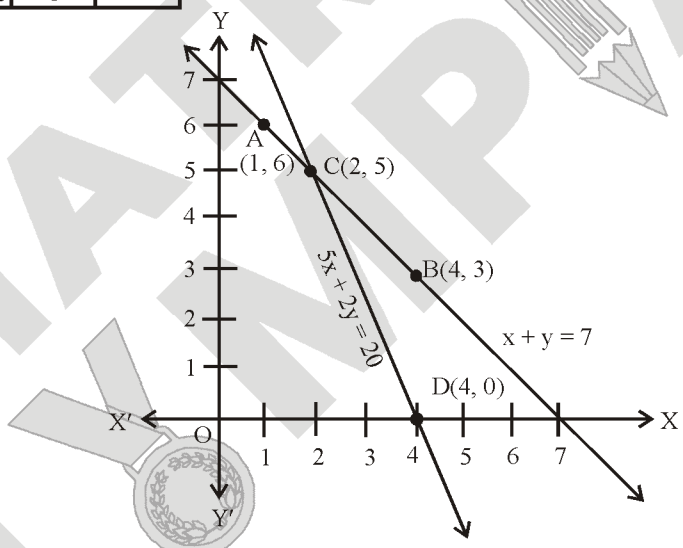
x	1	4
y	6	3
Points	A	B

Again, $5x + 2y = 20$

$$\Rightarrow y = \frac{20 - 5x}{2}$$

In tabular form :

x	2	4
y	5	0
Points	C	D



We plot the points A(1, 6), B(4, 3) and join them to form a line AB. Similarly, we plot the points C(2, 5), D(4, 0) and join them to get line CD. Clearly, the two lines intersect at the point C.

Example 2

By comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident :

(i) $5x - y + 7 = 0$
$10x - 2y + 15 = 0$

(ii) $3x + y - 14 = 0$
$2x + 5y - 5 = 0$

Solution :

(i) We have $5x - y + 7 = 0$, $10x - 2y + 15 = 0$

Here $a_1 = 5$, $b_1 = -1$, $c_1 = 7$,

$a_2 = 10$, $b_2 = -2$, $c_2 = 15$

Now, $\frac{a_1}{a_2} = \frac{5}{10} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-1}{-2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{7}{15}$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the lines are parallel.

(ii) We have $3x + y - 14 = 0$, $2x + 5y - 5 = 0$

Here $a_1 = 3$, $b_1 = 1$, $c_1 = -14$,

$a_2 = 2$, $b_2 = 5$, $c_2 = -5$

Now, $\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{1}{5}, \frac{c_1}{c_2} = \frac{-14}{-5} = \frac{14}{5}$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the lines intersect at a point.

Example 3

Find the value of p for which the pair of linear equations $2px + 3y = 7$, $2x + y = 6$ has exactly one solution.

Solution :

The given equations are $2px + 3y = 7$ and $2x + y = 6$ has exactly one solution i.e., they are intersecting lines.

We write these equations in standard form :

$2px + 3y - 7 = 0$

$2x + y - 6 = 0$

Here $a_1 = 2p$, $b_1 = 3$, $c_1 = -7$

$a_2 = 2$, $b_2 = 1$, $c_2 = -6$

$\frac{a_1}{a_2} = \frac{2p}{2} = p, \frac{b_1}{b_2} = \frac{3}{1}$.

Since the lines are intersecting therefore

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow p \neq 3$.

Hence, there will be a solution for all real values of p except 3.

2.2 ALGEBRAIC METHODS OF SOLVING A PAIR OF LINEAR EQUATIONS

A system of linear equations can be solved algebraically by following three methods :-

(1) Substitution method

(2) Elimination method

(3) Cross – multiplication method

Substitution Method

In this method we express one variable in terms of other in any one of the equations and substitute this in the other equation as follows :

Steps :

(i) Express one variable (say y) in terms of other variable (say x) from one of the given equations.

(ii) Substitute this value of y in the other equation to get a linear equation in one variable i.e., in terms of x, which can be solved.

(iii) Substitute the value of x obtained in step (ii) in the equation used in (i) to get the value of y.

Note : (i) We may interchange the role of x and y in the above steps.

(ii) Sometimes we may get statements with no variable. If the statement is true we can say that the equations has infinitely many solutions, otherwise we can say that the equations has no solutions.

Focus Point

- We have substituted the value of one variable by expressing it in terms of the other variable to solve the pair of linear equations thats why, the method is known as the substitution method.

Example 4

Solve the following pair of equations by substitution method.

$7x - 15y = 2$ and $x + 2y = 3$

Solution :

Here $7x - 15y = 2$ ...(1)

$x + 2y = 3$ ...(2)

From equation (2), $x = 3 - 2y$ ...(3)

Now, substituting the value of x in equation (1), we get

$7(3 - 2y) - 15y = 2 \Rightarrow 21 - 14y - 15y = 2$

$\Rightarrow \quad -29y = -19,\ y = 19/29$

Substituting value of y in equation (3), we get

$$x = 3 - 2\left(\frac{19}{29}\right) = \frac{49}{29}$$

Therefore, the solution is $x = \frac{49}{29}, y = \frac{19}{29}$

Example 5

Solve $2x + 3y - 9 = 0$ and $4x + 6y - 18 = 0$ by substitution method.

Solution :

Here $2x + 3y - 9 = 0$ ...(1)

$4x + 6y - 18 = 0$ ...(2)

From equation (1), $3y = 9 - 2x \Rightarrow y = \frac{9-2x}{3}$ ...(3)

Substituting y from equation(3) in equation(2), we get

$$4x + 6\left(\frac{9 - 2x}{3}\right) - 18 = 0$$

$\Rightarrow 4x + 2(9 - 2x) - 18 = 0 \Rightarrow 4x + 18 - 4x - 18 = 0 \quad \Rightarrow 4x - 4x + 18 - 18 = 0$

$\Rightarrow 0 = 0$, which is true statement.

Hence, the given pair of linear equations has infinitely many solutions. Let us find the solutions. Putting $x = k$ (any real constant) in equation(3), we get $y = 9 - 2k/3$

Hence, $x = k$, $y = 9 - 2k/3$ is the required solution where k is any real number.

Elimination method

Elimination method involves removing one of the variables by method discussed below.

Let $a_1x + b_1y + c_1 = 0$ .....(1)

$a_2x + b_2y + c_2 = 0$ .....(2)

be the given pair of linear equations.

Step 1 : Multiply the equations (1) and (2) both by some suitable non–zero constants to make the coefficients of one variable (x or y) numerically equal in both the equations.

Step 2 : Now add (or subtract) one equation to the other (or from the other) so that one variable gets eliminated.

First possibility

We get a linear equation in single variable x (or y)

It gives $x = x_1$ (say). Then proceed to step 3.

Second possibility

We get a true equation involving no variable. Therefore, the given pair of linear equation has no solution.

Step 3 : Substituting $x = x_1$ obtained in step 2 in either of the given equation (1) or (2) and to get the value of the second variable $y = y_1$ (say).

Thus, we get (x_1, y_1) as the unique solution of the given pair of linear equations.

Example 6

Solve for x and y :

$$\frac{x+1}{2}+\frac{y-1}{3}=8, \frac{x-1}{3}+\frac{y+1}{2}=9$$

Solution :

We have $\frac{x+1}{2}+\frac{y-1}{3}=8$

$\Rightarrow 3x + 3 + 2y - 2 = 48$

$\Rightarrow 3x + 2y = 47$ ...(1)

And $\frac{x-1}{3}+\frac{y+1}{2}=9$

$\Rightarrow 2x - 2 + 3y + 3 = 54$ $\Rightarrow 2x + 3y = 53$ ...(2)

Multiplying equation(1) by 2 and equation(2) by 3, we get

$6x + 4y = 94$ ...(3)

$6x + 9y = 159$ ...(4)

Subtracting equation(3) from equation(4), we get

$5y = 65 \Rightarrow y = 13$

Putting y = 13 in equation(1), we get

$3x + 2(13) = 47$

$\Rightarrow 3x = 47 - 26 \Rightarrow x = \frac{21}{3} = 7$

Hence, x = 7 and y = 13

Example 7

Solve : $9x - 4y = 8$ and $13x + 7y = 101$.

Solution :

$9x - 4y = 8$ .....(1)

$13x + 7y = 101$ .....(2)

Multiplying equation (1) by 7 and equation (2) by 4, we get

$63x - 28y = 56$ .....(3)

$52x + 28y = 404$ .....(4)

Adding equation (3) and equation (4), we get

$115x = 460 \Rightarrow x = \dfrac{460}{115} = 4$

Substitute x = 4 in equation (1), we have

$9(4) - 4y = 8 \Rightarrow 36 - 8 = 4y$

$\Rightarrow y = \dfrac{28}{4} = 7$

$\therefore x = 4, y = 7.$

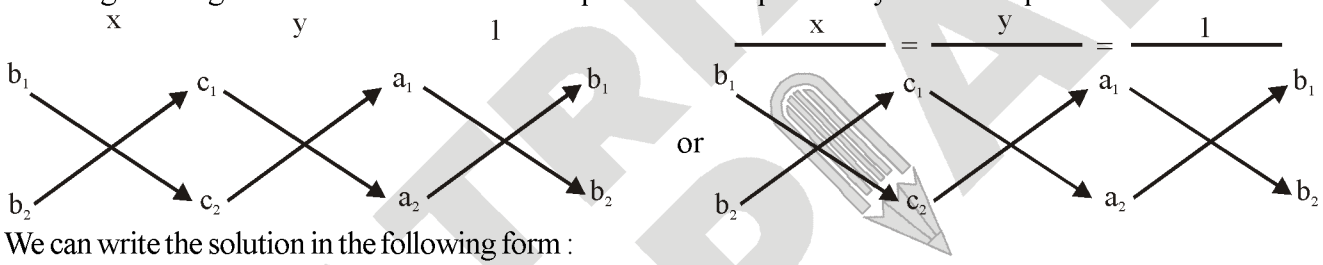
Cross-multiplication method

Consider a pair of linear equations in two variables in standard form as

$a_1x + b_1y + c_1 = 0$ ....(1)

$a_2x + b_2y + c_2 = 0$ ....(2)

Use the given diagram to find the solution of the pair of linear equations by cross multiplication method.



We can write the solution in the following form :

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Hence, we get $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$, where $a_1b_2 - a_2b_1 \neq 0$

Example 8

Solve the following system of equation by cross–multiplication method.

$2x + 3y + 8 = 0$

$4x + 5y + 14 = 0$

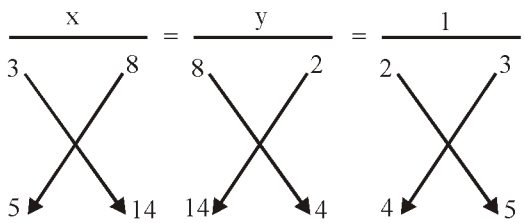
Solution :

The given system of equation is

$2x + 3y + 8 = 0$

$4x + 5y + 14 = 0$

By cross–multiplication, we get



$$\Rightarrow \frac{x}{3\times 14-5\times 8}=\frac{y}{4\times 8-2\times 14}=\frac{1}{2\times 5-4\times 3}$$

$$\Rightarrow \frac{x}{42-40}=\frac{y}{32-28}=\frac{1}{10-12}\Rightarrow \frac{x}{2}=\frac{y}{4}=\frac{1}{-2}$$

$$\Rightarrow \frac{x}{2}=-\frac{1}{2}\Rightarrow x=-1 \text{ and } \frac{y}{4}=-\frac{1}{2}\Rightarrow y=-2.$$

Example 9

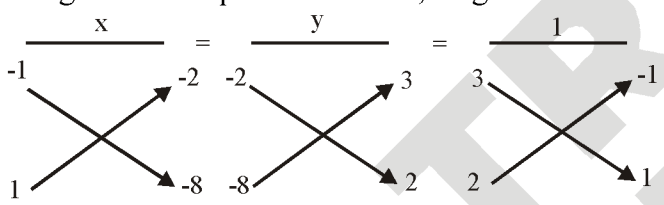
Solve $3x-y-2=0$, $2x+y-8=0$ by method of cross–multiplication.

Solution :

The given system of equation is

$$3x-y-2=0$$
$$2x+y-8=0$$

Using cross–multiplication method, we get



$$\Rightarrow \frac{x}{(8+2)}=\frac{y}{(-4+24)}=\frac{1}{(3+2)}\Rightarrow \frac{x}{10}=\frac{y}{20}=\frac{1}{5}$$

$$\Rightarrow \frac{x}{10}=\frac{1}{5} \text{ and } \frac{y}{20}=\frac{1}{5}\Rightarrow x=\frac{10}{5} \text{ and } y=\frac{20}{5}$$

$\therefore$ $x=2$ and $y=4$.

Hence, $x=2$ and $y=4$ is the required solution.

3. EQUATION REDUCIBLE TO A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

These equations are not linear but can be reduced to linear form by making some suitable substitutions. The illustrations will make it more clear.

Example 10

Solve the pair of equations :

$$\frac{2}{x}+\frac{3}{y}=13 \text{ and } \frac{5}{x}-\frac{4}{y}=-2$$

Solution :

Let us write the given pair of equations as

$$2\left(\frac{1}{x}\right)+3\left(\frac{1}{y}\right)=13 \qquad ....(1)$$

$$5\left(\frac{1}{x}\right)-4\left(\frac{1}{y}\right)=-2 \qquad ....(2)$$

These equations are not in the linear form.

However, if we substitute $\frac{1}{x}=p$ and $\frac{1}{y}=q$ in equation (1) and (2), we get

$2p + 3q = 13$ and $5p - 4q = -2$

So, we have expressed the equations as a pair of linear equations. Now, we can use any method to solve these equations and get $p = 2$, $q = 3$.

Then we put $q = \frac{1}{y}$ and $p = \frac{1}{x}$.

Substituting the values of p and q we get

$\frac{1}{x}=2$ i.e., $x = \frac{1}{2}$ and $\frac{1}{y}=3$ i.e., $y = \frac{1}{3}$.

Example 11

Solve the following pair of equations by reducing them to a pair of linear equations.

$$\frac{1}{x-1}+\frac{1}{y-2}=2;\frac{6}{x-1}-\frac{3}{y-2}=1$$

Solution :

Let us put $\frac{1}{x-1}=p$ and $\frac{1}{y-2}=q$. Then the given equations can be written as :

$p + q = 2$ and $6p - 3q = 1$

On solving these equations, we get $p = \frac{7}{9}$ and $q = \frac{11}{9}$.

Now, we have $\frac{1}{x-1}=\frac{7}{9} \Rightarrow 7x - 7 = 9 \Rightarrow 7x = 16 \Rightarrow x = \frac{16}{7}$

$\frac{1}{y-2}=\frac{11}{9} \Rightarrow 9 = 11y - 22 \Rightarrow y = \frac{31}{11}$ Hence, $x = \frac{16}{7}, y = \frac{31}{11}$.

Example 12

A boat goes 12 km upstream and 40 km downstream in 8 hours. It can go 16 km upstream and 32 km downstream in the same times. Find the speed of the boat in still water and the speed of the stream.

Solution :

Let the speed of the boat in still water be x km/hr, and the speed of the stream be y km/hr then speed of boat in downstream is $(x + y)$ km/hr and the speed of boat in upstream is $(x - y)$ km/hr.

Case (i) : Distance covered in upstream = 12 km

$\therefore$ Time $= \dfrac{12}{x-y}$ hr

Distance covered in downstream = 40 km

$\therefore$ Time $= \dfrac{40}{x+y}$ hr

Total time is 8 hrs. Therefore,

$$\frac{12}{x-y} + \frac{40}{x+y} = 8 \qquad .....(1)$$

Case (ii) : Distance covered in upstream = 16 km

$\therefore$ Time $= \dfrac{16}{x-y}$ hr

Distance covered in downstream = 32 km

$\therefore$ Time $= \dfrac{32}{x+y}$ hr

Total time is 8 hrs. Therefore,

$$\frac{16}{x-y} + \frac{32}{x+y} = 8 \qquad .....(2)$$

Putting $\dfrac{1}{x-y} = u$ and $\dfrac{1}{x+y} = v$ equation (1) and equation (2) becomes

$3u + 10v = 2$ .....(3)

$4u + 8v = 2$ .....(4)

Multiplying equation (3) by 4 and equation (4) by 5 and subtracting, we get

$-8u = -2 \qquad \therefore u = \dfrac{1}{4}$

Putting $u = \dfrac{1}{4}$ in equation (3), we get $v = \dfrac{1}{8}$

Now, $u = \dfrac{1}{4}$ and $v = \dfrac{1}{8}$

$$\Rightarrow \frac{1}{x-y} = \frac{1}{4} \Rightarrow x - y = 4 \qquad .....(5)$$

$$\Rightarrow \frac{1}{x+y} = \frac{1}{8} \Rightarrow x + y = 8 \qquad .....(6)$$

On solving equation (5) and equation (6), we get x = 6 and y = 2

Hence, speed of the boat = 6 km/hr and speed of the stream = 2 km/hr.

SOLVED EXAMPLES

SE. 1

Solve the following system of equations by using substitution :

$(a + b)\,x + (a - b)\,y = a^2 + b^2$

$(a - b)\,x + (a + b)\,y = a^2 + b^2$

Ans. The given system of equation are :

$(a + b)\,x + (a - b)\,y = a^2 + b^2$ ....(1)

$(a - b)\,x + (a + b)\,y = a^2 + b^2$ ....(2)

From (2), we get $(a+b)y = a^2 + b^2 - (a-b)x$

$$\Rightarrow y = \frac{(a^2+b^2)}{(a+b)} - \frac{(a-b)}{(a+b)}x \quad ....(3)$$

Substituting $y = \frac{(a^2+b^2)}{(a+b)} - \frac{(a-b)}{(a+b)}x$ in (1), we get

$$(a+b)x + (a-b)\left[\frac{(a^2+b^2)}{(a+b)} - \frac{(a-b)}{(a+b)}x\right] = a^2+b^2$$

$$\Rightarrow (a+b)x + \frac{(a-b)(a^2+b^2)}{a+b} - \frac{(a-b)^2}{a+b}x = a^2+b^2$$

$$\Rightarrow (a+b)x - \left(\frac{a^2+b^2-2ab}{a+b}\right)x$$

$$= (a^2+b^2) - \frac{(a-b)(a^2+b^2)}{a+b}$$

$$\Rightarrow (a+b)x - \left(\frac{a^2+b^2-2ab}{a+b}\right)x$$

$$= (a^2+b^2)\left[1 - \frac{a-b}{a+b}\right]$$

$$\Rightarrow \frac{(a^2+2ab+b^2)x - (a^2-2ab+b^2)x}{a+b}$$

$$= (a^2+b^2)\left(\frac{a+b-a+b}{a+b}\right)$$

$$\Rightarrow \frac{4ab}{a+b}x = \frac{(a^2+b^2)2b}{a+b}$$

$$\Rightarrow 4abx = 2b\,(a^2+b^2) \Rightarrow x = \frac{a^2+b^2}{2a}$$

Putting the value of x in (3), we get

$$y = \frac{a^2+b^2}{a+b} - \frac{(a-b)}{(a+b)} \times \frac{a^2+b^2}{2a}$$

$$\Rightarrow y = \frac{(a^2+b^2)}{a+b}\left[1 - \frac{(a-b)}{2a}\right]$$

$$= \left(\frac{a^2+b^2}{a+b}\right)\left(\frac{2a-a+b}{2a}\right)$$

$$\Rightarrow y = \left(\frac{a^2+b^2}{a+b}\right)\left(\frac{a+b}{2a}\right) \Rightarrow y = \frac{a^2+b^2}{2a}$$

Hence, the solution is $x = \frac{a^2+b^2}{2a}$ and $y = \frac{a^2+b^2}{2a}$.

SE. 2

The area of a rectangle increases by 76 square units, if the length and breadth is increased by 2 units. However, if the length is increased by 3 units, and breadth is decreased by 3 units, the area gets reduced by 21 square units. Find the length and breadth of the rectangle.

Ans. Let the length of the rectangle be x units and the breadth be y units.

Then, $(x + 2)(y + 2) = xy + 76$

$\Rightarrow 2x + 2y + 4 = 76$

$\Rightarrow x + y = 36$ ....(1)

In the second case : $(x + 3)(y - 3) = xy - 21$

$\Rightarrow 3y - 3x - 9 = -21$

$\Rightarrow 3x - 3y = 21 - 9 = 12$

$\Rightarrow x - y = 4$ ....(2)

From (1), $y = 36 - x$

Substituting the value of y in (2), we get

$x - [36 - x] = 4 \Rightarrow x - 36 + x = 4$

$\Rightarrow 2x = 40 \therefore x = 20$ units

and $y = 36 - 20 = 16$ units

Hence, length = 20 units and breadth = 16 units.

SE. 3

Solve the following system of equations by the method of cross–multiplication :

$\frac{a}{x} - \frac{b}{y} = 0, \frac{ab^2}{x} + \frac{a^2b}{y} = a^2 + b^2$; where $x \neq 0$, $y \neq 0$

Ans. The given system of equations are :

$\frac{a}{x} - \frac{b}{y} = 0$ ....(1)

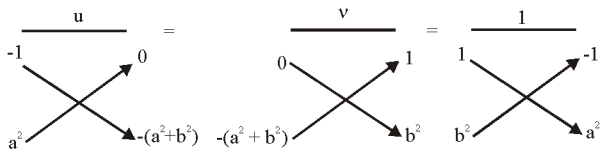
$\frac{ab^2}{x} + \frac{a^2b}{y} - (a^2 + b^2) = 0$ ....(2)

Putting $\frac{a}{x} = u$ and $\frac{b}{y} = v$ in equations (1) and (2)

system of equation reduces to

$u - v = 0$ and $b^2u + a^2v - (a^2 + b^2) = 0$

By the method of cross–multiplication, we have



$$\Rightarrow \frac{u}{a^2 + b^2} = \frac{v}{(a^2 + b^2)} = \frac{1}{a^2 + b^2}$$

$$\Rightarrow \frac{u}{a^2 + b^2} = \frac{1}{a^2 + b^2} \Rightarrow u = 1 \text{ and}$$

$$\frac{v}{(a^2 + b^2)} = \frac{1}{a^2 + b^2} \Rightarrow v = 1$$

Now, $u = \frac{a}{x} = 1 \Rightarrow x = a$

$v = \frac{b}{y} = 1 \Rightarrow y = b$

Hence, the solution of the given system of equations is $x = a$, $y = b$

SE. 4

Solve : $\frac{1}{2(2x + 3y)} + \frac{12}{7(3x - 2y)} = \frac{1}{2}$

$\frac{7}{2x + 3y} + \frac{4}{3x - 2y} = 2,$

where $2x + 3y \neq 0$ and $3x - 2y \neq 0$

Ans. Putting $\frac{1}{2x + 3y} = p$ and $\frac{1}{3x - 2y} = q$ the given equation becomes

$\frac{p}{2} + \frac{12q}{7} = \frac{1}{2} \Rightarrow 7p + 24q = 7$ ....(1)

and $7p + 4q = 2$ ....(2)

Subtracting equation (2) from equation (1), we get

$$20q = 5 \Rightarrow q = \frac{5}{20} = \frac{1}{4}$$

Putting $q = \frac{1}{4}$ in equation (2), we get

$$7p + 4\left(\frac{1}{4}\right) = 2 \Rightarrow 7p + 1 = 2$$

$$\Rightarrow 7p = 2 - 1 = 1 \Rightarrow p = \frac{1}{7}$$

Now, $p = \frac{1}{2x+3y} = \frac{1}{7} \Rightarrow 2x + 3y = 7$ ....(3)

$q = \frac{1}{3x-2y} = \frac{1}{4} \Rightarrow 3x - 2y = 4$ ....(4)

Multiplying equation (3) by 2 and equation (4) by 3, we get

$4x + 6y = 14$ ....(5)

$9x - 6y = 12$ ....(6)

On adding equation (5) and equation (6), we get

$13x = 26 \Rightarrow x = 2$

One putting $x = 2$ in equation (3), we get

$2(2) + 3y = 7 \Rightarrow 3y = 7 - 4 = 3 \Rightarrow y = 1$

Hence, the solution is $x = 2, y = 1$.

SE. 5

Solve: $2x^2 + 3y^2 = 35$; $\frac{x^2}{2} + \frac{y^2}{3} = 5$

Ans. Let $x^2 = u$, $y^2 = v$

$\Rightarrow 2u + 3v = 35$ and $\frac{u}{2} + \frac{v}{3} = 5$

$\Rightarrow 2u + 3v = 35$ ....(1)

and $3u + 2v = 30$ ....(2)

Multiply (1) by 3 and (2) and subtracting them, we get $6u + 9v - 6u - 4v = 105 - 60$

$\Rightarrow 5v = 45 \Rightarrow v = 9$

Substituting $v = 9$ in (1), we get $2u + 27 = 35$

$\Rightarrow 2u = 8 \Rightarrow u = 4 \quad \Rightarrow \quad x^2 = 4, y^2 = 9$

$\therefore x = \pm 2, y = \pm 3$ is the required solution.

EXERCISE – I

ONLY ONE CORRECT TYPE

1. The line represented by the equation $3x - 4y = 12$ cuts x-axis at the point.

(A) (0, 3) (B) (3, 0)
(C) (4,0) (D) (–4, 0)

2. The line represented by the point equation $4x + 5y - 20 = 0$ cuts y-axis at the point.

(A) (0, –5) (B) (0, –4)
(C) (0, 4) (D) (0, 5)

3. Which of the following point lies on the line represented by $2x + 7y = 19$?

(A) (–1, 3) (B) (1, –3)
(C) (–3, 5) (D) (5, 2)

4. The value of a for which $x = 2$, $y = -1$ is a solution of the equation $3ax + 5ay = 2$ is

(A) –2 (B) 3
(C) –1 (D) 2

5. The abscissa of a point lying on $y - 2x = 3$ is –3, then its ordinate will be -

(A) –3 (B) –2
(C) 2 (D) –1

6. The points (–2, 0) and (3, 0) lie on

(A) y-axis
(B) x-axis
(C) line parallel to y-axis
(D) line $3x - 9 = 0$

7. The distance of point (–3, 4) from origin is

(A) 3 units (B) 4 units
(C) 5 units (D) 2 units

8. The lines $4x + 6 = 0$ and $x = -\frac{3}{2}$ are

(A) parallel (B) perpendicular
(C) meet at origin (D) none of these

9. The lines $2(x + 3) = 3(x + 9)$ and $y(a + 1) = 2(y - 5)$ are

(A) parallel
(B) perpendicular
(C) both parallel to x-axis
(D) both parallel to y-axis

10. The graph of $y = -9$ is a line parallel to

(A) x-axis (B) y-axis
(C) $x = -9$ (D) none of these

11. An equation of the type $ax + by + c = 0$, where $a \neq 0$, $b \neq 0$, $c = 0$ represent a line which passes through

(A) (2, 4) (B) (0, 0)
(C) (3, 2) (D) none of these

12. The graph of $2x + 1 = 0$ and $3y - 9 = 0$ intersect at the point

(A) $\left(\frac{1}{2}, -3\right)$ (B) $\left(-\frac{1}{2}, -3\right)$
(C) $\left(-\frac{1}{2}, 3\right)$ (D) none of these

13. An equation of the type $ax + by + c = 0$, where $a = 0$, $b \neq 0$, $c \neq 0$ represent a line parallel to

(A) x-axis (B) y-axis
(C) $x + y = 0$ (D) none of these

14. The area of the triangle formed by $x + y = 10$ and the coordinate axis is

(A) 50 sq. units (B) 25 sq. units
(C) 40 sq. units (D) none of these

15. The distance between $x = \pm 20$ is

(A) 20 units (B) 30 units
(C) 40 units (D) none of these

16. Ram's age is thrice of Shib's age. After 10 years Shib's age will be half of Ram's age, then difference of their ages is

(A) 1year (B) 10 years
(C) 15 years (D) 20 years

17. $3x + 7y = 61$ and $11x + y = 51$, then $(y - x) =$

(A) 3 (B) 4
(C) 5 (D) 6

18. In crossing a distance of 30 km. A takes 2 hrs more than B. If A doubles his speed, he would have taken 1 hr less than B. Then their speeds are

(A) 5 km/hr and 7.5 km/hr
(B) 5.5 km/hr and 7.5 km/ hr
(C) 4.5 km/ hr and 5.5 km/hr
(D) none of these

19. A boat goes upstream 30 km and downstream 44 km in 10 hours. It also goes upstream 40 km and downstream 55 km in 13 hours. Then the speed of stream and that of boat are

(A) 5 km/hr and 3 km/hr
(B) 8 km/hr and 3.5 km/hr
(C) 8 km/hr and 3 km/hr
(D) none of these

20. The two lines $y = mx$ and $y = 2mx$ intersect at the point

(A) (m, 2m) (B) (2m, m)
(C) (0, 0) (D) none of these

21. The value of x in $\frac{x}{x+1} = -1$ is

(A) 0 (B) $-\frac{1}{3}$
(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

22. The graph of $x + 2y = 5$ and $x = 1$ intersect at the point

(A) (1, 2) (B) (2, 1)
(C) (1, 0) (D) none of these

23. The point $A\left(-3m, \frac{1}{m}\right)$, $m > 0$ lies in

(A) 1st quadrant (B) 2nd quadrant
(C) 3rd quadrant (D) 4th quadrant

24. The graph of line $y = k$, is

(A) perpendicular to x-axis
(B) parallel to y-axis
(C) perpendicular to y-axis
(D) none of these

25. The equation of x-axis is

(A) $y = 0$ (B) $x = 0$
(C) $x + y = 0$ (D) none of these

PARAGRAPH TYPE

PASSAGE # I

The graph of the linear equation $ax + by + c = 0$ is a straight line. If the equation does not contain the constant term, then the graph of equation will pass through origin. If (x_1, y_1) is a solution of the equation $ax + by + c = 0$, then it will also lie on the graph of the equation and vice-versa. Based on the above passage, answer the following questions.

26. Graph of the equation $3x - 7y = 0$

(A) Passes through the origin

(B) Does not pass through the origin

(C) Is a straight line parallel to x-axis

(D) Is a straight line parallel to y-axis

27. If the point P(2a – 1, a – 2) lies on the graph of the equation $x - y = -\frac{1}{2}$, then the value of a is

(A) $-\frac{1}{2}$ (B) $-\frac{3}{2}$

(C) $-\frac{5}{2}$ (D) $-\frac{7}{2}$

28. The definite solution of the equation $ax - by = 0$ is

(A) (0, 0) (B) (a, b)

(C) (0, – 1) (D) (– 1, 0)

PASSAGE–II : The system of linear equations is given as $4x - (3k + 2)y = 20$ $(11k - 3)x - 10y = 40$, where $k \neq 0$. Based on the above equations, answer the following questions.

29. If the given system of linear equations has infinitely many solutions, then the value of k is

(A) 0 (B) 1

(C) –1 (D) 2

30. If k = 4, then the set of linear equations has

(A) No solution (B) Unique solution

(B) Infinite solution (D) Data insufficient

31. If k = 0, then the given system of linear equations has

(A) No solution (B) Unique solution

(B) Infinite solution (D) Data insufficient

MATCH THE COLUMN TYPE

32. Match the following :

Column – I	Column – II
(P) The value of k for which (2, 1) is a solution of the equation $3x + 2y = k$ is	(i) 4
(Q) Equation of a straight line at an angle of 45° with the positive x-axis is	(ii) 8
(R) If (a, – 2) is a solution of $x + y = 2$, then the value of a is	(iii) y-axis
(S) The graph of the equation $x = 4$ is parallel to	(iv) $y = x$

(A) P – (ii), Q – (iv), R – (i), S – (iii)

(B) P – (iv), Q – (ii), R – (i), S – (iii)

(C) P – (ii), Q – (iii), R – (iv), S – (i)

(D) P – (i), Q – (iv), R – (ii), S – (iii)

33. In European countries temperature is measured in Fahrenheit, whereas in Asian countries, it is measured in Celsius. The linear equation that converts Fahrenheit to Celsius is

$$C = \frac{1}{9}(F - 32) \times 5.$$

Match the temperatures given in List – I with temperatures given in List – II.

Column – I	**Column – II**
(P) 26° C in F =	(i) 104°
(Q) 64° F in C =	(ii) 8.9°
(R) 48° F in C =	(iii) 78.8°
(S) 40° C in F =	(iv) 17.8°

(A) P – (iv), Q – (i), R – (iii), S – (ii)

(B) P – (iii), Q – (iv), R – (ii), S – (i)

(C) P – (iv), Q – (iii), R – (i), S – (ii)

(D) P – (ii), Q – (i), R – (iii), S – (iv)

Space for Notes :

EXERCISE – II

VERY SHORT ANSWER TYPE

1. Find the value of 'a' so that $x = -3$, $y = 2$ is a solution of the equation $4ax + 9(2 + ay) = 0$.

2. Find the four solutions of the equation

   $3(x + 2y) - (x + y) + 13 = 0$.

3. Plot the points A (0, 2), B(–3, 2), C (–3, 5) and D (0, 5) and join AB, BC, CD and DA. What figure do you obtain ?

4. Draw a rectangle whose sides are rerpresented by the straight lines $x = -1$, $x = 5$, $y = 2$ and $y = -3$.

5. Draw the graph of the following equations using the same pair of axes:

   (i) $y = 3x + 5$
   (ii) $3x - y = 7$

   Are these lines parallel ?

6. Draw the graphs of $x + y = 3$ and $3x - 2y = 4$. Also find the coordinates of the points where the two lines intersect.

7. Express y in terms of x, it being given that

   $2x - 3y + 11 = 0$. Check whether the point

   (–4, 1) lies on the line represented by the equation $2x - 3y + 11 = 0$.

8. A square is formed by the lines $x = 2$, $x = 6$, $y = 5$, $y = 9$. A circle is inscribed in it. Find the centre of the circle.

9. Show that $x = m + l$, $y = m + l$ is a solution of the equation $(x + y)(m^2 + l^2) - 2(m^3 + l^3) = ml(x+y)$.

10. If the point (k, k^2) lies on the graph of the equation $x + y = 6$, then find the value/values of k.

SHORT ANSWER TYPE

1. Check whether $x = \dfrac{a^2 + ab + b^2}{a + b}$, $y = -\dfrac{ab}{a + b}$ is solution of the equation $ax + by = a^2$.

2. Prove that the triangle formed by the lines $y = 0$, $4x + 3y = 12$ and $4x - 3y + 12 = 0$ is an isosceles triangle.

3. If $\left(\dfrac{1}{a}, b\right)$ lies on the equation $2ax + 4y = 2$, then find the value of b.

4. The cost of 6 cows are same as cost of 8 goats. If cost of 9 cows and twice numbered goats is Rs. 9000, then find the cost of 3 cows and 6 goats ?

5. A and B are two cities 100 km apart. From these two cities two cars start moving in same direction and they meet after 5 hours. If they moved towards each other then they would meet after 1 hour. What were the speeds of the cars ?

LONG ANSWER TYPE

1. The denominator of a fraction is 4 more than twice the numerator. When both the numerator and the denominator are decreased by 6, then the denominator becomes 12 times the numerator. Determine the fraction.

2. The sum of the digits of a two digit number is 8. If 18 is added to the number, then the resultant number is equal to number obtained by reversing the digits of the original number. Find the original number.

3. Find the value(s) of k for which the system of equation $kx - y = 2$, $6x - 2y = 3$ has.

   (i) a unique solution

   (ii) no solution

4. Ten years ago, father was twelve times as old as his son and ten years hence, he will be twice as old as his son will be. Find their present age.

5. Solve $\frac{2}{x+y} - \frac{1}{x-y} = 11$ and

$\frac{5}{x+y} - \frac{4}{x-y} = 8.$

TRUE / FALSE TYPE

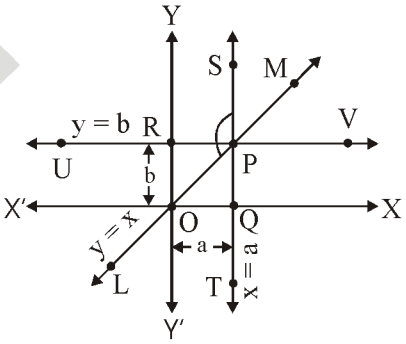
1. A linear equation 2x + 3y = 5 has a unique solution.
2. All the points (2, 0), (–3, 0), (4, 2) and (0, 5) don't lie on the x-axis.
3. The graph of the equation y = mx + c passes through the origin.
4. Every point on the graph of a linear equation in two variables does not represent a solution of the linear equation.
5. The graph of every linear equation in two variables need not be a line.

FILL IN THE BLANKS

1. a : b = 7 : 3 and a + b = 30, then b = _____.
2. If x + y = k, x – y = n and k > n , then y is ____. (positive/negative)
3. The graph of x = – 2 is a line parallel to the ___ . (x-axis/y-axis)
4. The grpah of the equation 3x + 5y = 7 is a ____ line. (straight/vertical/horizontal line)
5. The pair of equations x = –3 & y = 7 graphically represents _____ lines. (parallel/intersecting)

ANALYTICAL PROBLEMS & BRAIN TEASER

1. Draw a quadrilateral whose sides are represented by graphs of the equation x = 0, y = 0, 2y – 3x – 1 = 0 and 5x – y – 10 = 0. Determine the coordinates of the vertices of the quadrilateral.
2. Draw the graphs of y = – 9 and x – 2 = y and find the point of intersection, if any.
3. The coordinates of the point A are (x, y), where x < 0 and y > 0. If the line segment OA (O is the origin) makes an angle of 150° with the positive x-axis, then by what angle (anticlockwise) should OA be retated so as to make x positive and y negative ?
4. The graph of the equation y = x, x = a and y = b intersect each other at point P as shown in the figure. Find the value of ∠SPO.



5. If p and q are whole numbers, then find the number of ordered pairs (p, q) which satisfy the equation 2p + 3q = 25.

NUMERICAL PROBLEMS

1. If the system of equations $2x + 3y = 5$,

   $4x + ky = 10$ has infinitely many solutions, then find k.

2. The difference between a two digit number and the number obtained by interchanging the digits is 27. What is the difference between the two digits of the number.

3. If the system of equations $4x + 6y = 7$,

   $4ax + 2(a + b)y = 28$ has infinitely many solutions, if $b = ka$. Find value of k.

4. A boat covers 24 km upstream and 36 km downstream in 6 hours while it covers 36 km upstream and 24 km downstream in $6\frac{1}{2}$ hours. The speed of the current is ___ km/hr.

5. By solving equations $3x + 4y = 25$ and $4x + 3y = 24$ with help of cross-multiplication method we obtain $\frac{x}{a} = \frac{y}{b} = \frac{1}{c}$. Then find the value of $\frac{b^2 - a^2}{c}$.

Space for Notes :

Answer Key

EXERCISE–I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	C	A	D	A	B	C	D	B	A	B	C	A	A	C
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
D	A	A	C	C	D	A	B	C	A	A	B	A	B	B
31	32	33												
B	A	B												

EXERCISE II

VERY SHORT ANSWER TYPE

1. a = –3 **2.** (1, –3); (–4, –1); (6, –5); (–9, 1) **3.** a square **5.** yes **6.** (2,1)

7. $y = \frac{2x+11}{3}$; (–4, 1) lies on the line **8.** (4, 7) **10.** k = 2, –3

SHORT ANSWER TYPE

1. yes it is a solution. **3.** 0 **4.** Rs. 3000 **5.** 60 km/h and 40 km/h

LONG ANSWER TYPE

1. required fraction = $\frac{7}{18}$ **2.** number is 35 **3.** (i) if $k \neq 3$, (ii) k = 3

4. Father = 34 years and son = 12 years **5.** $x = \frac{25}{312}, y = \frac{1}{312}$.

TRUE / FALSE

1. False 2. True 3. False 4. False 5. False

FILL IN THE BLANKS

1. 9 2. negative 3. y-axis 4. straight 5. intersecting

ANALYTICAL PROBLEMS & BRAIN TEASER

1. $\left(0,\frac{1}{2}\right),(0,0),(2,0),(3,5)$ **2.** (– 7, – 9) **3.** Between 120° and 210° **4.** 135° **5.** 4

NUMERICAL PROBLEMS

1. 6 **2.** 3 **3.** 2 **4.** 2 **5.** 49

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : PAIR OF LINEAR EQUATIONS IN TWO VARIABLES)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

QUADRATICS EQUATIONS 4

Concepts

INTRODUCTION

We are already familiar with the application of linear equations and system of linear equations in solving problems related to our day to day life. However, to solve some problem we require the application of second degree equation. In this chapter, We shall discuss equations in one variable in which the highest of the variable is two, known as quadratic equations.

1. QUADRATIC POLYNOMIAL

A polynomial of degree two is called a quadratic polynomial. E.g. $x^2 + 4$, $x^2 - 5x + 6$, $x^2 + \sqrt{3}x$, $\sqrt{2}x^2 + 2x - 6$.

A quadratic polynomial can have at most three terms namely, terms containing x^2, x and constant.

The general format of a quadratic polynomial in x is $ax^2 + bx + c$, where a, b, c are numbers and $a \neq 0$.

In quadratic polynomial $f(x) = ax^2 + bx + c$; a, b, c the called coefficients.

If for $x = \alpha$, where α is a real number, the value of quadratic expression becomes zero, then α is called zero of $ax^2 + bx + c$

A quadratic polynomial has at most two zeros.

Note : $f(x) = ax^2 + bx + c$ is also called quadratic expression.

2. POLYNOMIAL EQUATION

If f(x) be a polynomial, then $f(x) = 0$ is called a polynomial equation. Let f(x) be a quadratic polynomial, then $f(x) = 0$ is called quadratic equation. The general expression forming a quadratic equation is $ax^2 + bx + c = 0$ where a, b, $c \in R$ and $a \neq 0$.

2.1 QUADRATIC EQUATION

The second degree polynomial equations are commonly known as quadratic equation. *i.e.*, if P(x) is a quadratic polynomial, then $P(x) = 0$ is called a quadratic equation. The general form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

Case I : When $b \neq 0$, $c \neq 0$, then quadratic equation is of the type $ax^2 + bx + c = 0$.

Case II : When $b \neq 0$, $c = 0$, then quadratic equation is of the type $ax^2 + c = 0$.

Case III : When $b \neq 0$, $c = 0$, then quadratic equation is of the type $ax^2 + bx = 0$.

Case IV : When $b = 0$, $c = 0$, then quadratic equation is of the type $ax^2 = 0$.

2.2 ZEROS OF A QUADRATIC EQUATION

Zeros of a quadratic equation : Zeros of quadratic polynomial $ax^2 + bx + c$ where a, b, c are real number and $a \neq 0$ is found by solving the corresponding equation $ax^2 + bx + c = 0$ called a quadratic equation.

If the real number α and β are two zeros of the quadratic polynomial $ax^2 + bx + c$, then α and β are roots of the quadratic equation $ax^2 + bx + c = 0$.

There will be two roots for a quadratic equation and can be found by solving the equation $ax^2 + bx + c = 0$.

Roots are also called solutions of $ax^2 + bx + c = 0$.

Example 1

Find whether

(i) $x = 2$ is a zero of $x^2 - 5x + 6$.

(ii) $x = -4$, $x = -1$ are zeros of $x^2 + 5x + 4$

(iii) $x = -\frac{1}{3}, x = -\frac{1}{4}$ are zeros of $6x^2 + 5x + 1$

Solution :

(i) $x = 2$

$\therefore x^2 + 5x + 6 = 2^2 - 5(2) + 6 = 4 - 10 + 6 = 0$

$\therefore x = 2$ is a zero of $x^2 - 5x + 6$.

(ii) $x = -4$

$\therefore x^2 + 5x + 4 = (-4)^2 + 5(-4) + 4$

$= 16 - 20 + 4 = 0$

$\therefore x = -4$ is a zero of $x^2 + 5x + 4$

$x = -1$

$\therefore x^2 + 5x + 4 = (-1)^2 + 5(-1) + 4 = 1 - 5 + 4 = 0$

$\therefore x = -1$ is zero of $x^2 + 5x + 4$

$\therefore x = -4$ and $x = -1$ are zeros of $x^2 + 5x + 4$.

(iii) $x = -\frac{1}{3}, \quad \therefore 6x^2 + 5x + 1 = 6\left(-\frac{1}{3}\right)^2 + 5\left(-\frac{1}{3}\right) + 1$

$= 6 \times \frac{1}{9} - \frac{5}{3} + 1 = \frac{5}{3} - \frac{5}{3} = 0$

$x = -\frac{1}{4}, \therefore 6x^2 + 5x + 1 = 6\left(-\frac{1}{4}\right)^2 + 5\left(-\frac{1}{4}\right) + 1 = \frac{2}{16} \neq 0$

$\therefore x = -\frac{1}{3}$ is a zero and $x = -\frac{1}{4}$ is not a zero of $6x^2 + 5x + 1$.

Example 2

If $x = 2$ and $x = 3$ are roots of equation $3x^2 - 2kx + 2m = 0$. Find the value of k and m.

Solution :

Since $x = 2$ and $x = 3$ are roots at the equation $3x^2 - 2kx + 2m = 0$

and $3(3)^2 - 2k(3) + 2m = 0$

$12 - 4k + 2m = 0$ and $27 - 6k + 2m = 0$

Solving these two equations, we get $k = \frac{15}{2}$ and $m = 9$

3. SOLUTION OF QUADRATIC EQUATION

The zeroe's of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are same and called the solution of the quadratic equation. We can find the solution of quadratic equation by the following methods as explained below.

3.1 BY FACTORISATIO METHOD

Let $ax^2 + bx + c = 0$; $a \neq 0$ be a quadratic equation. Let it be expression into two linear factor $(px + q)$ and $(rx + s)$ where $p, q, r, s \in R$ such that $p \neq 0$ and $r \neq 0$, then $ax^2 + bx + c = (px + q)(rx + s) = 0$

$\Rightarrow$ $px + q = 0$ or $rx + s = 0$

$\Rightarrow$ $x = -\frac{q}{p}$ and $x = -\frac{s}{r}$

Example 3

Solve $81x^2 - 64 = 0$

Solution :

$81x^2 - 64 = 0$

$\Rightarrow$ $(9x)^2 - (8)^2 = 0$

$\Rightarrow$ $(9x + 8)(9x - 8) = 0$

$\therefore$ $x = -\frac{8}{9}$ or $x = \frac{8}{9}$

$\therefore$ $x = -\frac{8}{9}, \frac{8}{9}$ are solutions of $81x^2 - 64 = 0$

Example 4

Solve $5x^2 - 7x - 6 = 0$

Solution :

$5x^2 - 7x - 6 = 0$

$\Rightarrow$ $5x^2 - 10x + 3x - 6 = 0$

$\Rightarrow$ $5x(x - 2) + 3(x - 2) = 0$

$\Rightarrow$ $(x - 2)(5x + 3) = 0$

$\Rightarrow$ $x - 2 = 0$ or $5x + 3 = 0$

$\Rightarrow$ $x = 2$ or $x = -\frac{3}{5}$ $\therefore$ $x = 2, -\frac{3}{5}$ are solution of $5x^2 - 7x - 6 = 0$

Example 5

Solve $\frac{x+1}{x-1}+\frac{x-2}{x+2}=3$ $(x \neq 1, -2)$

Solution :

$\frac{x+1}{x-1}+\frac{x-2}{x+2}=3 \Rightarrow \frac{x+1}{x-1}+\frac{x-2}{x+2}-3=0$

$\Rightarrow \frac{(x+1)(x+2)+(x-1)(x-2)-3(x-1)(x+2)}{(x-1)(x+2)}=0$

$\Rightarrow (x+1)(x+2)+(x-1)(x-2)-3(x-1)(x+2)=0$

$\Rightarrow -x^2-3x+10=0 \qquad \Rightarrow -(x^2+3x-10)=0$

$\Rightarrow x^2+3x-10=0$

$\Rightarrow (x+5)(x-2)=0 \qquad \Rightarrow x=-5$, or $x=2$

$\therefore$ $x=-5, 2$ are the solutions of the given equation.

3.2 BY COMPLETING THE SQUARE METHOD

In this method, we rewrite a quadratic equation in the form $(x+\alpha)^2=c^2$. This method is called the method of completing the perfect square, where c is a constant term. Following is a method to obtain the roots of the equation by using method of completing squares.

Let the quadratic equation : $ax^2+bx+c=0$

Dividing throughtout by a $(a \neq 0)$, we get $x^2+\frac{b}{a}x+\frac{c}{a}=0$

By adding and subtracting the square of $\left(\frac{1}{2}\text{coefficient of } x\right)^2$.

We get,

$$x^2+2.\left(\frac{b}{2a}\right)x+\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2+\frac{c}{a}=0$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

If $b^2-4ac \geq 0$, then $\sqrt{b^2-4ac}$ is a real number

$$\therefore \left(x+\frac{b}{2a}\right)=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$\therefore\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (This is also called quadratic formula)

Note : Here $D = b^2 - 4ac$ is called discriminant of the quadratic equation $ax^2 + bx + c = 0$

Example 6

Find the roots of quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$ by the method of completing of square.

Solution :

We have, $4x^2 + 4\sqrt{3}x + 3 = 0$

Divide both sides by '4', we get $x^2 + \sqrt{3}x + \dfrac{3}{4} = 0$

$\Rightarrow\ x^2 + \sqrt{3}x = -\dfrac{3}{4}$

Adding $\left(\dfrac{1}{2}\text{coefficient of } x\right)^2$ *i.e.* $\left(\dfrac{\sqrt{3}}{2}\right)^2$ on both sides

$\Rightarrow\ x^2 + \sqrt{3}x + \left(\dfrac{\sqrt{3}}{2}\right)^2 = -\dfrac{3}{4} + \left(\dfrac{\sqrt{3}}{2}\right)^2$

$\Rightarrow\ \left(x + \dfrac{\sqrt{3}}{2}\right)^2 = \pm 0$

$\Rightarrow\ x = -\dfrac{\sqrt{3}}{2}, -\dfrac{\sqrt{3}}{2}$

Hence , roots of equation are $-\dfrac{\sqrt{3}}{2}$ and $-\dfrac{\sqrt{3}}{2}$.

Example 7

Find the roots of $a^2x^2 - 3abx + 2b^2 = 0$ by method of completing square.

Solution :

We have, $a^2x^2 - 3abx + 2b^2 = 0$

$\Rightarrow\ x^2 - 3\dfrac{b}{a}x + 2\dfrac{b^2}{a^2} = 0$ [Divide both side by a^2]

$\Rightarrow\ x^2 - 3\dfrac{b}{a}x + \left(\dfrac{3b}{2a}\right)^2 = -\dfrac{2b^2}{a^2} + \left(\dfrac{3b}{2a}\right)^2$

$\Rightarrow\ \left(x - \dfrac{3b}{2b}\right)^2 = -\dfrac{2b^2}{a^2} + \dfrac{9b^2}{4a^2}$

$$\Rightarrow \left(x-\frac{3b}{2b}\right)^2=\frac{b^2}{4a^2} \qquad \Rightarrow \qquad x-\frac{3b}{2a}=\pm\frac{b}{2a}$$

$$\Rightarrow \quad x-\frac{3b}{2a}=\frac{b}{2a} \text{ or } x-\frac{3b}{2a}=-\frac{b}{2a}$$

$$\Rightarrow \quad x=\frac{3b}{2a}+\frac{b}{2a} \text{ or } x=\frac{3b}{2a}-\frac{b}{2a}$$

$$\Rightarrow \quad x=\frac{2b}{a} \text{ or } x=\frac{b}{a}$$

Example 8

Solve $abx^2 + (b^2 - ac)x - bc = 0$ by using quadratic formula.

Solution :

We have, $abx^2 + (b^2 - ac)x - bc = 0$

Comparing it with $Ax^2 + Bx + C = 0$, we have

$A = ab$, $B = b^2 - ac$, $C = -bc$

$D = B^2 - 4AC = (b^2 - ac)^2 - 4(ab)(-bc)$

$= b^4 + a^2c^2 - 2ab^2c + 4ab^2c \qquad \Rightarrow \qquad (b^2 + ac)^2 > 0$

So, the roots of the given equation are real and given by

$$\alpha=\frac{-B+\sqrt{D}}{2A}=\frac{-(b^2-ac)+\sqrt{(b^2+ac)^2}}{2ab}$$

$$\frac{-b^2+ac+b^2+ac}{2ab}=\frac{2ac}{2ab}=\frac{c}{d}$$

$$\beta=\frac{-B-\sqrt{D}}{2A}=\frac{-b^2+ac-b^2-ac}{2ab}=\frac{-2b^2}{2ab}=-\frac{b}{a}$$

$$\therefore \quad x=\frac{c}{b},\frac{-b}{a}$$

Example 9

The sum of squares of two consecutive positive integers is 221. Find the integers.

Solution :

Let x be one of the positive integers. Then the other is $x + 1$

$\therefore$ Sum of squares of the integers $= x^2 + (x + 1)^2 = 221$

$\therefore \quad x^2 + x^2 + 2x + 1 - 221 = 0$

$2x^2 + 2x - 220 = 0$

$2(x^2 + x - 110) = 0$

$\therefore$ $(x - 10)(x + 11) = 0$ $\therefore$ $x = 10$ or $x = -11$

$\therefore$ consecutive positive integers are 10 and –11.

Example 10

A takes 6 days less than the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time taken by B to finish the work.

Solution :

Let B takes x days to complete the work. Then A takes (x – 6) days to do the same work.

$\Rightarrow \dfrac{1}{x-6} + \dfrac{1}{x} = \dfrac{1}{4}$

$\Rightarrow \dfrac{x + x - 6}{(x-6)x} = \dfrac{1}{4}$ $\Rightarrow \dfrac{2x-6}{x^2-6x} = \dfrac{1}{4}$

$x^2 - 6x = 8x - 24$ $\Rightarrow$ $x^2 - 14x + 24 = 0$

$\Rightarrow x^2 - 12x - 2x + 24 = 0$

$\Rightarrow (x - 2)(x - 12) = 0$

$\Rightarrow x = 2$ or $x = 12$

But x can't be 2, so x = 2 is rejected take x = 12

Hence, B alone can finish the work in 12 days.

4. NATURE OF ROOTS

We have already learnt that the roots of a quadratic equation with real coefficients can be real or complex. When the roots are real, they can be rational or irrational and, also, they can be equal or unequal.

Let α, β be the two roots of the quadratic equation $ax^2 + bx + c = 0$ and we have the quadratic formula to find α, β, *i. e.*, $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$ and $b^2 - 4ac > 0$

since $b^2 - 4ac$ determines nature of roots of equation $ax^2 + bx + c = 0$, hence it called discriminant of the quadratic equation denoted by D.

So, a quadratic equation $ax^2 + bx + c = 0$ has

(i) complex conjugates roots, when $D < 0$, means no real roots.

(ii) rational and equal roots, when $D = 0$

(iii) rational and unequal roots, when $D > 0$ and a perfect square

(iv) irrational and uneuqal roots, when $D > 0$ and not a perfect square

Example 11

Find the nature of roots of following equations :

(i) $x^2 + 3x + 2$ (ii) $x^2 + 5x + 7$

Solution :

(i) Here, $D = b^2 - 4ac = 9 - (4 \times 1 \times 2) = 1$, Hence $D > 0$ so roots are real

(ii) Here, $D = b^2 - 4ac = 25 - (4 \times 1 \times 7) = -3$, Hence $D < 0$ so roots of the equation are not real, means roots will be complex conjugates

Example 12

Find the values of p for which the quadratic equation $6x^2 + px + 6 = 0$ has real roots.

Solution :

$D = b^2 - 4ac = p^2 - 4 \times 6 \times 6 = p^2 - 144$

As the equation has real roots $D \geq 0$

$\therefore\ p^2 - 12^2 \geq 0 \quad \Rightarrow (p + 12)(p - 12) \geq 0$ .....(1)

(1) holds good if

(i) $p + 12 \geq 0$ and $p - 12 \geq 0$

$\therefore\ p \geq -12,\ p \geq 12$

$\therefore\ p \geq 12$ or $P \geq -12$

(ii) $p + 12 \leq 0$ and $p - 12 \leq 0$

$\therefore\ p \leq -12$ and $p \leq 12$ $\quad \therefore\ p \leq -12$

$\therefore$ Required values of p are $p \leq -12$ or $p \geq 12$

Example 13

Determine whether the following quadratic equation have real roots and if they have, find them.

(i) $2x^2 + 11x - 6 = 0$

(ii) $x^2 - 6x + 9 = 0$

(iii) $x^2 + x + 1 = 0$

(iv) $x^2 - 4x - 9 = 0$

Solution :

(i) $2x^2 + 11x - 6 = 0$

Comparing this equation with $ax^2 + bx + c = 0$

We have $a = 2$, $b = 11$, $c = -6$

$\therefore$ Discriminant $D = b^2 - 4ac = 11^2 - 4(2)(-6) = 121 + 48 = 169$

$\therefore$ $D > 0$, given equation will have two distinct real roots say α, β

given by $\alpha = \dfrac{-b+\sqrt{b^2-4ac}}{2a} = \dfrac{-11+\sqrt{169}}{4} = \dfrac{-11+13}{4} = \dfrac{1}{2}$

$\beta = \dfrac{-b-\sqrt{b^2-4ac}}{2a} = \dfrac{-11-\sqrt{169}}{4} = \dfrac{-11-13}{4} = -6$

$\therefore$ the two roots are $\dfrac{1}{2}$ and -6

(ii) $x^2 - 6x + 9 = 0$

$\therefore$ $a = 1, b = -6, c = 9$

$\therefore$ $D = b^2 - 4ac = (-6)^2 - 4(1)(9) = 36 - 36 = 0$

$\therefore$ Equation has a repeated root given by $\alpha = -\dfrac{b}{2a} = -\dfrac{(-6)}{2\times 1} = 3$

(iii) $x^2 + x + 1 = 0$

$a = 1, b = 1, c = 1$

$\therefore$ $D = b^2 - 4ac = 1^2 - 4(1)(1) = -3 < 0$

$\therefore$ The equation does not have real roots.

(iv) $x^2 - 4x - 9 = 0$

$a = 1, b = -4, c = -9$

$D = b^2 - 4ac = (-4)^2 - 4(1)(-9) = 16 + 36 = 52 > 0$

$\therefore$ The equation has two roots given by

$\dfrac{4+\sqrt{52}}{2}, \dfrac{4-\sqrt{52}}{2}$ $\quad\therefore\quad x = \dfrac{4+\sqrt{52}}{2}, \dfrac{4-\sqrt{52}}{2}$

$\dfrac{4+2\sqrt{13}}{2}, \dfrac{4-2\sqrt{13}}{2}$ *i.e.* $2+\sqrt{13}, 2-\sqrt{13}$ are the required solutions.

5. RELATION BETWEEN ROOTS AND COEFFICIENTS OF A QUADRATIC EQUAITON

If α, β are roots fo $ax^2 + bx + c = 0$ then

$\alpha = \dfrac{-b+\sqrt{b^2-4ac}}{2a}, \beta = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$

$\therefore$ $\alpha + \beta = \text{sum of roots} = -\dfrac{b}{a} = -\dfrac{(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$

$\therefore$ $\alpha\beta = \text{product of roots} = \dfrac{c}{a} = \dfrac{\text{constant term}}{\text{coefficient of } x^2}$

Focus Point

(i) A quadratic equation whose roots are α and β is given by $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e. x^2 – (sum of roots)x + product of roots = 0

(ii) Some important formulae :

1. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
2. $(\alpha^3 + \beta^3) = (\alpha + \beta)^3 - 3\alpha\beta\ (\alpha + \beta)$
3. $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$
4. $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
5. $\alpha^2 - \beta^2 = (\alpha - \beta)\ (\alpha + \beta) = (\alpha + \beta)\ \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$
6. $\alpha^3 - \beta^3 = (\alpha - \beta)\ (\alpha^2 + \alpha\beta + \beta^2)$
7. $\alpha^4 - \beta^4 = (\alpha + \beta)\ (\alpha - \beta)\ (\alpha^2 + \beta^2) = (\alpha + \beta)\ (\alpha - \beta)\ [(\alpha + \beta)^2 - 2\alpha\beta]$
8. $\alpha^5 + \beta^5 = (\alpha^3 + \beta^3)\ (\alpha^2 + \beta^2) - \alpha^2\beta^2\ (\alpha + \beta)$

Example 14

If α, β are roots of $ax^2 + bx + c = 0$. Find the quadratic equation whose roots are.

(i) $2\alpha,\ 2\beta$ (ii) $\alpha + 3,\ \beta + 3$ (iii) $\frac{\alpha}{4}, \frac{\beta}{4}$ (iv) $\frac{1}{\alpha}, \frac{1}{\beta}$

Solution :

α, β are roots of $ax^2 + bx + c = 0$

$$\alpha + \beta = -\frac{b}{a},\ \alpha\beta = \frac{c}{a}$$

(i) We have to find the equation whose roots are 2α, 2β

$\therefore$ for the required equation

$$\text{sum of roots} = 2\alpha + 2\beta = 2(\alpha + \beta) = 2\left(-\frac{b}{a}\right) = -\frac{2b}{a}$$

$$\text{product of roots} = (2\alpha)\ (2\beta) = 4\alpha\beta = 4\left(\frac{c}{a}\right) = \frac{4c}{a}$$

$\therefore$ The equation whose roots are 2α, 2β is.

x^2 – (sum of roots)x + (product of roots) = 0

$\therefore\ x^2 + \frac{2b}{a}x + \frac{4c}{a} = 0$ *i.e.* $ax^2 + 2bx + 4c = 0$

(ii) Sum $= \alpha + 3 + \beta + 3 = \alpha + \beta + 6 = -\frac{b}{a} + 6 = \frac{-b+6a}{a}$

product $= (\alpha + 3)(\beta + 3) = \alpha\beta + 3\alpha + 3\beta + 9$

$= \alpha\beta + 3(\alpha + \beta) + 9 = \frac{c}{a} - \frac{3b}{a} + 9 = \frac{c - 3b + 9a}{a}$

$\therefore$ required equation is $x^2 -$ (sum of roots)x + product of roots $= 0$

$$x^2 - \left(\frac{-b+6a}{a}\right)x + \left(\frac{c-3b+9a}{a}\right) = 0$$

i.e. $ax^2 - (-b + 6a)x + (c - 3b + 9a) = 0$

(iii) sum $= \frac{\alpha}{4} + \frac{\beta}{4} = \frac{\alpha+\beta}{4} = \frac{-b/a}{4} = \frac{-b}{4a}$

Product $= \frac{\alpha}{4} \times \frac{\beta}{4} = \frac{c}{16a}$

$\therefore$ Required equation is $x^2 - \left(-\frac{b}{4a}\right)x + \frac{c}{16a} = 0$ *i.e.* $16ax^2 + 4bx + c = 0$

(iv) sum $= \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-b/a}{c/a} = -\frac{b}{c}$

product $= \frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{c/a} = \frac{a}{c}$

$\therefore$ required equation is $x^2 - \left(-\frac{b}{c}\right)x + \frac{a}{c} = 0$

i.e. $cx^2 + bx + a = 0$

Example 15

If sum of the roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero, then prove that the product of roots is $-\frac{1}{2}(a^2 + b^2)$

Solution :

We have $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$

$\Rightarrow \quad x^2 + (a + b - 2c)x + (ab - bc - ca) = 0$

Let α, β be the roots of the equation.

Given $\alpha + \beta = 0 \quad \Rightarrow -(a + b - 2c) = 0 \quad \Rightarrow c = \frac{a+b}{2}$ .....(i)

$\alpha\beta = ab - bc - ca = ab - c(a+b) = ab - \frac{(a+b)^2}{2}$ Using equation (i)

$= \frac{2ab-(a+b)^2}{2} = -\frac{1}{2}(a^2+b^2)$

Example 16

For the quadratic equation $ax^2 + bx + x = 0$; $a \neq 0$ find the condition that :

(i) one root is reciprocal of the other

(ii) one root is n times the other root.

Solution :

We have $ax^2 + bx + c = 0$; $a \neq 0$

Let α, β be root of the equation then $\alpha + \beta = -\frac{b}{a}$ .....(i)

and $\alpha\beta = \frac{c}{a}$ .....(ii)

(i) Let $\beta = \frac{1}{\alpha}$, then $\alpha\beta = 1$

$\Rightarrow \frac{c}{a} = 1$ or $c = a$, which is required condition

(ii) Let $\beta = n\alpha$

Now from equation (i) and (ii)

$\alpha + n\alpha = -\frac{b}{a}$ and $\alpha.n\alpha = \frac{c}{a}$

$\alpha = \frac{-b}{a(n+1)}$ and $n\alpha^2 = \frac{c}{a}$

$\Rightarrow n\left[\frac{-b}{a(n+1)}\right]^2 = \frac{c}{a}$ $\Rightarrow \frac{nb^2}{a^2(n+1)^2} = \frac{c}{a}$

$\Rightarrow ac(n+1)^2 = b^2n$

Which is the required condition

6. FORMATION OF A QUADRATIC EQUATION FROM GIVEN ROOTS

If α, β be the roots of a quadratic equation then the quadratic equation is given by:

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$ *i.e.*

$x^2 - (\text{sum of roots})x + \text{product of roots} = 0$

SOLVED EXAMPLES

SE. 1

Deepak and sudhir together have 26 marbles. Hoth of them lost 3 marbles each, and the product of the number of marbles they now have is 91. We want to find out how many marbles they had to start with. Represent the above problem mathematically in terms of a quadratic equation.

Ans. Let the number of marbles Deepak had = x, Then the number of marbles Sudhir had in the beginning = (26 – x).

[$\because$ Deepak and Sudhir together have 26 marbles]

The number of marbles left with Deepak after losing 3 marbles = (x – 3)

The number of marbles left with Sudhir after losing 3 marbles = (26 – x – 3) = (23 – x)

According to question, we have (x – 3) (23 – x) = 91

$\Rightarrow$ $23x - x^2 - 69 + 3x = 91$

$\Rightarrow$ $-x^2 + 26x - 69 - 91 = 0$

$\Rightarrow$ $-x^2 + 26x - 160 = 0$

$\Rightarrow$ $x^2 - 26x + 160 = 0$

Hence, the required quadratic equation is

$x^2 - 26x + 160 = 0$

SE. 2

A train travels a distance of 720 km at a uniforms speed. If the speed has been 12 km/hr less, then it would have taken 2 hrs more to cover the same distance. Represent the above problem mathematically in terms of a quadratic equation.

Ans. Let the uniform speed of the train be x km/hr.

Time taken to travel $720\,\text{km} = \dfrac{720}{x}$ hrs

When the speed is reduced by 12 km/hr, the time taken to travel $720\,\text{km} = \dfrac{720}{(x-12)}$ hrs

According to question, $\dfrac{720}{(x-12)} - \dfrac{720}{x} = 3$

$\Rightarrow$ $720 \times \left\{\dfrac{1}{(x-12)} - \dfrac{1}{x}\right\} = 3$

$\Rightarrow$ $720 \times \left\{\dfrac{x-(x-12)}{x(x-12)}\right\} = 3$

$\Rightarrow$ $720 \times \dfrac{12}{x(x-12)} = 3$

$\Rightarrow$ $720 \times 12 = 3 \times x\,(x - 12)$

$\Rightarrow$ $x(x - 12) = \dfrac{720 \times 12}{3}$

$\Rightarrow$ $x^2 - 12x = 720 \times 4$

$\Rightarrow$ $x^2 - 12x - 2880 = 0$ is the required quadratic equation

SE. 3

If roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\dfrac{a}{b} = \dfrac{c}{d}$.

Ans. $(a^2 + b^2)x^2 - 2(ac + bd)x + c^2 + d^2 = 0$

Here $A = a^2 + b^2$, $B = -2(ac + bd)$, $C = c^2 + d^2$

$\because$ Roots are equal $\quad \therefore D = 0 \Rightarrow B^2 - 4AC = 0$

$\Rightarrow$ $4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2) = 0$

$\Rightarrow$ $4[a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2] = 0$

$\Rightarrow$ $-4[a^2d^2 + b^2c^2 - 2abcd] = 0$

$\Rightarrow$ $(ad - bc)^2 = 0 \Rightarrow ad = bc \Rightarrow \dfrac{a}{b} = \dfrac{c}{d}$

SE. 4

Solve for x : $4x^2 - 4a^2x + (a^4 - b^4) = 0$.

Ans. We have, $4x^2 - 4a^2x + (a^4 - b^4) = 0$.

$\Rightarrow$ $(2x)^2 - 2(2x)(a^2) + (a^2)^2 - b^4 = 0$

$\Rightarrow$ $(2x - a^2)^2 - (b^2)^2 = 0$

$\Rightarrow$ $(2x - a^2 + b^2)(2x - a^2 - b^2) = 0$

$[a^2 - b^2 = (a + b)(a - b)]$

If $2x - a^2 + b^2 = 0$, then $x = \frac{a^2 - b^2}{2}$

and if $2x - a^2 - b^2 = 0$, then $x = \frac{a^2 + b^2}{2}$

Hence, $x = \frac{a^2 - b^2}{2}, \frac{a^2 + b^2}{2}$

SE. 5

Solve : $5^{(x+1)} + 5^{(2-x)} = 5^3 + 1$

Ans. We have, $5^{(x+1)} + 5^{(2-x)} = 5^3 + 1$

$\Rightarrow 5^x . 5 + 5^2 . 5^{-x} = 126 \quad \Rightarrow 5^x . 5 + \frac{25}{5^x} = 126$

Let $5^x = y$

$\Rightarrow 5y + \frac{25}{y} = 126$

$\Rightarrow 5y^2 - 126y + 25 = 0$

$\Rightarrow 5y^2 - 125y - y + 25 = 0$

$\Rightarrow 5y(y - 25) - 1(y - 25) = 0$

$\Rightarrow (y - 25)(5y - 1) = 0$

If $y - 25 = 0$, then $y = 25$, *i. e.* $5^x = 25$

or $5^x = 5^2 \quad \therefore x = 2$

If $5y - 1 = 0$, then $y = \frac{1}{5}$, *i. e.* $5^x = 5^{-1}$

$\therefore x = -1$

Hence, $x = 2, -1$

SE. 6

Solve for

$$x : \frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} + \frac{1}{(x-3)(x-4)} = \frac{1}{6}$$

Ans. we have,

$$\left[\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} + \frac{1}{(x-3)(x-4)} = \frac{1}{6}\right]$$

$$\Rightarrow \frac{x-3+x-1}{(x-1)(x-2)(x-3)} + \frac{1}{(x-3)(x-4)} = \frac{1}{6}$$

$$\Rightarrow \frac{2x-4}{(x-1)(x-2)(x-3)} + \frac{1}{(x-3)(x-4)} = \frac{1}{6}$$

$$\Rightarrow \frac{2}{(x-1)(x-3)} + \frac{1}{(x-3)(x-4)} = \frac{1}{6}$$

$$\Rightarrow \frac{2x-8+x-1}{(x-1)(x-3)(x-4)} = \frac{1}{6}$$

$$\Rightarrow \frac{3(x-3)}{(x-1)(x-3)(x-4)} = \frac{1}{6}$$

$$\Rightarrow \frac{3}{(x-1)(x-4)} = \frac{1}{6}$$

$\Rightarrow (x - 1)(x - 4) = 18 \quad \Rightarrow x^2 - 5x + 4 = 18$

$\Rightarrow x^2 - 5x - 14 = 0$

Using middle term splitting we get

$x^2 - 7x + 2x - 14 = 0$

$\Rightarrow x(x - 7) + 2(x - 7) = 0$

$\Rightarrow (x - 7)(x + 2) = 0$

If $x - 7 = 0$, then $x = 7$ and if $x + 2 = 0$, then $x = -2$

Hence, $x = 7, -2$.

SE. 7

Solve for x, using quadratic formula.

(i) $abx^2 + (b^2 - ac)x - bc = 0$

(ii) $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

Ans. (i) $abx^2 + (b^2 - ac)x - bc = 0$

Here, $A = ab$, $B = b^2 - ac$, $C = -bc$

$\therefore D = B^2 - 4AC = (b^2 - ac)^2 - 4(ab)(-bc)$

$= b^4 - 2b^2ac + a^2c^2 + 4ab^2c$

$= b^4 + 2b^2ac + a^2c^2 = (b^2 + ac)^2$

$$x = \frac{-B \pm \sqrt{D}}{2A} = \frac{-(b^2 - ac) \pm \sqrt{(b^2 + ac)^2}}{2ab}$$

$$\Rightarrow x = \frac{-(b^2 - ac) + (b^2 + ac)}{2ab} \text{ or } \frac{-(b^2 - ac) - (b^2 + ac)}{2ab}$$

$$\Rightarrow x = \frac{2ac}{2ab} \text{ or } x = \frac{-2b^2}{2ab} \quad \therefore x = \frac{c}{b} \text{ or } \frac{-b}{a}$$

(ii) $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

Here $A = 9$, $B = -9(a + b)$, $C = 2a^2 + 5ab + 2b^2$

$\therefore D = B^2 - 4AC = [-9(a+b)]^2 - 4.9(2a^2 + 5ab + 2b^2)$

$= 9[9a^2 + 18ab + 9b^2 - 8a^2 - 20ab - 8b^2]$

$= 9[a^2 - 2ab + b^2] = [3(a - b)]^2$

$\therefore$ Root are given by

$$x = \frac{-B \pm \sqrt{D}}{2A} = \frac{9(a+b) \pm 3(a-b)}{18}$$

$$\Rightarrow \quad x = \frac{9(a+b)+3(a-b)}{18} \quad \text{or}$$

$$x = \frac{9(a+b)-3(a-b)}{18}$$

$$\Rightarrow \quad x = \frac{12a+6b}{18} \quad \text{or} \quad x = \frac{6a+12b}{18}$$

$$\therefore \quad x = \frac{2a+b}{3} \quad \text{or} \quad x = \frac{a+2b}{3}$$

SE. 8

Find roots of following equations.

(i) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}; (x \neq 2, 4)$

(ii) $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}; (x \neq 3, -5)$

Ans. (i) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}; (x \neq 3, -5)$

$$\Rightarrow \quad \frac{(x-1)(x-4)+(x-2)(x-3)}{(x-2)(x-4)} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{x^2-5x+4+x^2-5x+6}{x^2-6x+8} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{2(x^2-5x+5)}{x^2-6x+8} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{x^2-5x+5}{x^2-6x+8} = \frac{5}{3}$$

$\Rightarrow$ $5x^2 - 30x + 40 = 3x^2 - 15x + 15$

$\Rightarrow$ $2x^2 - 15x + 25 = 0$

$\Rightarrow$ $2x^2 - 10x - 5x + 25 = 0$

$\Rightarrow$ $2x(x - 5) - 5(x - 5) = 0$

$\Rightarrow$ $(x - 5)(2x - 5) = 0$

$\Rightarrow$ $x - 5 = 0$ or $2x - 5$

$\Rightarrow$ $x = 5$ or $x = \frac{5}{2}$

$\therefore$ Required roots are $\frac{5}{2}$ and 5.

(ii) $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}; (x \neq 3, -5)$

$$\Rightarrow \quad \frac{8}{x^2+2x-15} = \frac{1}{6}$$

$$\Rightarrow \quad \frac{(x+5)-(x-3)}{(x-3)(x+5)} = \frac{1}{6}$$

$$\Rightarrow \quad \frac{8}{x^2+2x-15} = \frac{1}{6}$$

$\Rightarrow$ $x^2 + 2x - 15 = 48$

$\Rightarrow$ $x^2 + 2x - 63 = 0$

$\Rightarrow$ $x^2 + 9x - 7x - 63 = 0$

$\Rightarrow$ $x(x + 9) - 7(x + 9) = 0$

$\Rightarrow$ $(x + 9)(x - 7) = 0$

$\Rightarrow$ $x + 9 = 0$ or $x - 7 = 0$

$\Rightarrow$ $x = -9$ or $x = 7$

$\therefore$ Roots of given equation are –9 and 7

SE. 9

If – 4 is a root of the quadratic equation $x^2 + px - 4 = 0$ and the quadratic equation $x^2 + px + k = 0$ has equal roots, find the value of k.

Ans. Since – 4 is a root of the equation

$x^2 + px - 4 = 0$

$\therefore$ $(-4)^2 + p \times (-4) - 4 = 0$

[$\because$ A root always satisfies the equation]

$\Rightarrow$ $16 - 4p - 4 = 0$

$\Rightarrow$ $4p = 12$

$\Rightarrow$ $p = 3$

The equation $x^2 + px + k = 0$ has equal roots. Here $a = 1$, $b = p$ and $c = k$

$\therefore$ Discriminant $= 0$

$\Rightarrow$ $p^2 - 4k = 0$

$\Rightarrow$ $9 - 4k = 0$

$\Rightarrow$ $k = \frac{9}{4}$

SE. 10

If the sum of first n even natural numbers is 420, find the value of n.

Ans. We have 2 + 4 + 6 + 8 + ... to n terms = 420

$\Rightarrow \frac{n}{2}[2\times 2+(n-1)\times 2]=420$

$\Rightarrow n(2+n-1)=420$

$\Rightarrow n(n+1)=420$

$\Rightarrow n^2+n-420=0$

$\Rightarrow n^2+21n-20n-420=0$

$\Rightarrow n(n+21)(n-20)=0$

$\Rightarrow n=20, -21$

$\because$ n is a natural number

$\therefore n>0 \qquad \therefore \qquad n=20$

SE. 11

A swimming pool is filled with three pipes with uniform flow. The first two pipes operating simultaneously, fil the pool in same time during which the pool is filled by the third pipe alone. The second pipe fills pool the five hours faster than the first pipe and four hours slower than the third pipe. Find the time required by each pipe to fill the pool separately.

Ans. Let V be the volume of the pool and x be the number of hours required by the second pipe alone to fill the pool. Then, the first pipe takes (x + 5) hours, while the third pipe takes (x – 4) hours to fill the pool. So, the parts of the pool filled by the first, second and third pipes in one hour are respectively

$\frac{V}{x+5}, \frac{V}{x}$ and $\frac{V}{x-4}$

Let the time taken by the first and second pipes to fill the pool simultaneously be t hours. Then, the third pipe alos takes the same time to fill the pool.

$\therefore \left(\frac{V}{x+5}+\frac{V}{x}\right)t = \text{Volume of the pool} = \frac{V}{x-4}t$

$\Rightarrow \left(\frac{V}{x+5}+\frac{V}{x}\right)t=\frac{V}{x-4}t$

$\Rightarrow \frac{1}{x+5}+\frac{1}{x}=\frac{1}{x-4}$

$\Rightarrow \frac{x+x+5}{(x+5)x}=\frac{1}{x-4}$

$\Rightarrow (2x+5)(x-4)=x^2+5x$

$\Rightarrow x^2-8x-20=0$

$\Rightarrow x^2-10x+2x-20=0$

$\Rightarrow x(x-10)+2(x-10)=0$

$\Rightarrow (x-10)(x+2)=0$

$\Rightarrow x=10$ or $x=-2$. But, x cannot be negative.

So, x = 10

Hence, the timings required by first, second and third pipes to fill the pool individually are 15 hours, 10 hours and 6 hours respectively.

EXERCISE – I

ONLY ONE CORRECT TYPE

1. If one root of quadratic equation $2x^2 + kx - 6 = 0$ is 2 then the other root is.
(A) –1 (B) 2
(C) $-\frac{3}{2}$ (D) $\frac{3}{2}$

2. The roots of $4x^2 - 4ax + (a^2 - b^2) = 0$ are.
(A) a + b, a – b (B) $\frac{a+b}{2}, \frac{a-b}{2}$
(C) a + b, ab (D) $\frac{a+b}{2}$, ab

3. The roots of $x^2 + \left(a + \frac{1}{a}\right)x + 1 = 0$ are $(a \neq 0)$
(A) $a, \frac{1}{a}$ (B) $-a, -\frac{1}{a}$
(C) $a, -\frac{1}{a}$ (D) $-a, \frac{1}{a}$

4. The value of k if $(k + 1)x^2 - 2(k - 1)x + 1 = 0$ has equal roots.
(A) 0, 3 (B) 1, 2
(C) –3, –2 (D) 3, 2

5. If one root of the equation $5x^2 + 13x + k = 0$ is reciprocal of the other root then k is
(A) 6 (B) $\frac{1}{6}$
(C) 0 (D) 5

6. If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal the.
(A) a,b, c are in A.P.
(B) a, c, b are in A.P.
(C) b,a, c in A.P.
(D) none of these

7. If α, β are roots of $x^2 - 7x + 12 = 0$ then the equation whose roots are 3α, 3β is.
(A) $(3x)^2 - 7(3x) + 12 = 0$
(B) $\left(\frac{x}{3}\right)^2 - 7\left(\frac{x}{3}\right) + 12 = 0$
(C) $(x - 3)^2 - 7(x - 3) + 12 = 0$
(D) $(x + 3)^2 - 7(x + 3) + 12 = 0$

8. If α, β are roots of $2x^2 - 3x - 6 = 0$ then equation whose roots are $\alpha^2 + 2, \beta^2 + 2$ is.
(A) $4x^2 + 49x + 118 = 0$
(B) $4x^2 - 49x + 118 = 0$
(C) $4x^2 - 49x - 118 = 0$
(D) $x^2 - 49x + 118 = 0$

9. Which of the following is quadratic equation ?
(A) $x^2 + 3x + 4 = 0$
(B) $x^3 - 2x^2 + 4 = 0$
(C) $x + \frac{3}{x} = x^2$
(D) $2x^3 - x + 2 = x^2 + 4x - 4$

10. If the product of the roots of $ax^2 + bx + a^2 + 1 = 0$ is –2 then a is
(A) 2 (B) 1
(C) –2 (D) –1

11. If both the roots of the equation $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$ are real and equal then.
(A) a + b + c = 0 (B) a = b = c
(C) a + b = 2c (D) $b^2 = ac$

12. The value of $\sqrt{20 + \sqrt{20 + \sqrt{20 + ...\infty}}}$ is.
(A) 4 (B) –5
(C) 5 (D) 20

13. The number of real roots of $2x^4 + 5x^2 + 3 = 0$ is
(A) 3 (B) 2
(C) 4 (D) 0

14. If α,β are roots of the quadratic equation $kx^2 + 4x + 4 = 0$, then the value of k such that $\alpha^2 + \beta^2 = 24$, is.

(A) 1 (B) $-\frac{2}{3}$

(C) –1 (D) none of these

15. The area of a right-angled triangle is 30 m^2. If the base exceeds the altitude by 7 units, then the length of base is.

(A) 12 (B) 13

(C) 6 (D) 5

16. The quadratic equation whose root is $\frac{1}{2+\sqrt{5}}$ is

(A) $x^2 - 4x - 1 = 0$ (B) $x^2 - 4x + 1 = 0$

(C) $x^2 + 4x - 1 = 0$ (D) $x^2 + 4x + 1 = 0$

17. If $ax^2 + bx + c$ is a perfect square then $b^2 =$

(A) 2ac (B) ac

(C) 4ac (D) $\sqrt{2ac}$

18. The value of k so that the quadratic equation, $x^2 - 2x(1 + 3k) + 7(3 + 2k) = 0$ has equal roots is.

(A) 2 (B) 3

(C) 4 (D) 5

19. $ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$ has one root as 1 then ab =

(A) 3 (B) 3.5

(C) 6 (D) –3

20. If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, then.

(A) $\frac{a}{d} = \frac{b}{c}$ (B) $\frac{a^2+b^2}{c^2} = \frac{b^2+c^2}{d^2}$

(C) $\frac{a}{b} = \frac{c}{d}$ (D) none of these

21. If a and c are such that the quadratic equation $ax^2 - 5x + c = 0$ has 10 as the sum of the roots and also as the products of the roots, then the value of a is.

(1) 5 (2) $\frac{1}{2}$

(3) –5 (4) $-\frac{1}{2}$

22. The value of k such that the sum of the squares of the roots of the quadratic equation $x^2 - 8x + k = 0$ is 40.

(A) 10 (B) 11

(C) 12 (D) –12

23. If – 4 is a root of the equation $x^2 + px - 4 = 0$ and the equation $x^2 + px + q = 0$ has equal roots, then the value of q is.

(A) 4 (B) 5

(C) $\frac{4}{9}$ (D) $\frac{9}{4}$

24. If α and β are the roots of $x^2 - 5x + 4 = 0$, the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$ is.

(A) $\frac{4}{27}$ (B) $-\frac{27}{4}$

(C) $\frac{20}{27}$ (D) none of these

25. If α and β are the roots of the equation $3x^2 + 8x + 2 = 0$, then the value of $\alpha^2 + \beta^2$ is.

(A) $\frac{9}{52}$ (B) $-\frac{9}{52}$

(C) $\frac{52}{9}$ (D) none of these

PARAGRAPH TYPE

PASSAGE # I

To represent word problem in the form of quadratic equations, suppose the unknown required quantity can be taken as some variable x (say) and express the given condition in the form of x to form an equation in x. Then we express the equation in the descending powers of x. Thus standard quadratic equation is $ax^2 + bx + c = 0$, $a \neq 0$.

26. The product of two consecutive even integers is 128. Form the quadratic equation to find the integers.

(A) $x^2 + 2x - 128 = 0$

(B) $x^2 + x - 128 = 0$

(C) $x^2 - 2x - 128 = 0$

(D) $x^2 + 2x = -128$

27. A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/hr more, then it would have taken 2 hours less to cover the same distance. The quadratic equation is :

(A) $x^2 + 8x + 480 = 0$

(B) $x^2 + 8x - 1920 = 0$

(C) $x^2 - x + 200 = 0$

(D) $x^2 + 8x - 480 = 0$

28. Sandeep's father is 30 years older than him. The product of their ages 2 years from now will be 400. To find Sandeep's present age, the equation is :

(A) $x^2 + 9x - 13 = 0$

(B) $x^2 + 32x + 400 = 0$

(C) $x^2 + 34x - 336 = 0$

(D) $x^2 - 34x + 90 = 0$

PASSAGE # II

Roots of the quadratic equation of the type $(ax + b)(cx + d) = 0$ are given by the linear equations $ax + b = 0$ and $cx + d = 0$.

29. Find roots of $16x^2 - 9 = 0$.

(A) $x = \frac{4}{3}$ (B) $x = \frac{4}{3}, \frac{-4}{3}$

(C) $x = \frac{2}{3}, \frac{-2}{3}$ (D) $x = \frac{-3}{4}, \frac{3}{4}$

30. Find roots of $a^2x^2 - (a^2b^2 + 1)x + b^2 = 0$.

(A) $x = a, b$ (B) $x = \frac{1}{a^2}, b^2$

(C) $x = a^2b^2, b^2$ (D) $x = \frac{1}{b^2}, a^2$

31. Find roots of the quadratic equation $3x^2 - 2\sqrt{6}x + 2 = 0$.

(A) $x = \frac{9}{4}, \frac{3}{2}$ (B) $x = \frac{4}{3}, \frac{-9}{8}$

(C) $x = \sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$ (D) $x = \pm\sqrt{\frac{2}{3}}$

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from List - I and List - II are given as options (A), (B), (C) and (D) out of which one is correct.

32. Match the quadratic equations formed in List - I to that in List - II.

List – I	List – II
(P) The product of two consecutive even integers is 528.	(i) $z^2 - 45z + 500 = 0$
(Q) Megha and Latika have 45 chocolates. Both of them lost 3 each, and the product of the chocolates now is 374.	(ii) $x^2 - 15x - 50 = 0$
(R) The hypotenuse of rightangled triangle is 6 more than the shortest side and third side is 3 less than the hypotenuse.	(iii) $n^2 + 2n - 528 = 0$
(S) Difference between two numbers is 5 and the sum of their reciprocals is 1/10.	(iv) $y^2 - 6y - 27 = 0$

(A) (P) → (iii), (Q) → (iv), (R) → (i), (S) → (ii)

(B) (P) → (iii), (Q) → (i), (R) → (iv), (S) → (ii)

(C) (P) → (i), (Q) → (iii), (R) → (ii), (S) → (iv)

(D) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)

33. List - II gives roots of quadratic equations given in List - I match them correctly.

List – I	List – II
(P) $6x^2 + x - 12 = 0$	(i) – 6, 4
(Q) $8x^2 + 16x + 10 = 202$	(ii) 9, 36
(R) $x^2 - 45x + 324 = 0$	(iii) 3, – 1/2
(S) $2x^2 - 5x - 3 = 0$	(iv) – 3/2, 4/3

(A) (P) → (iv), (Q) → (i), (R) → (ii), (S) → (iii)

(B) (P) → (iv), (Q) → (ii), (R) → (i), (S) → (iii)

(C) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)

(D) (P) → (i), (Q) → (iii), (R) → (ii), (S) → (iv)

EXERCISE – II

VERY SHORT ANSWER TYPE

1. What is the nature of roots of the quadratic equation $4x^2 - 12x + 9 = 0$?
2. Write a quadratic equation, sum of whose zeroes is $2\sqrt{3}$ and product is 2.
3. The sum and product of the zeroes of a quadratic polynomial are $-\frac{1}{2}$ and -3 respectively. What is the quadratic equation.
4. What is nature of roots of the quadratic equation $3x^2 - 4\sqrt{3} + 4 = 0$.
5. Find the value of k so that the following quadratic equation has equal roots $2x^2 - (k-2)x + 1 = 0$.
6. What is the nature of the roots quadratic equation $2x^2 + 5x + 5 = 0$?
7. Find the roots of the quadratic equation $x^2 + 7x + 12 = 0$.
8. For what values of k the quadratic equation $9x^2 - 24x + k = 0$ has equal roots.
9. Find the value of k for which the quadratic equation $x^2 + 5kx + 16 = 0$ has no real roots.
10. Two numbers differ by 3, and their products is 504. Find the numbers.

SHORT ANSWER TYPE

1. Find the quadratic equation one of whose roots is $2 + \sqrt{5}$.
2. For what value of k, $(4-k)x^2 + (2k+4)x + (8k+1)$ is perfect square.
3. Solve by factorization $8x^2 - 22x - 21 = 0$.
4. If one root is equal to the square of the other root of the equation $x^2 + x - k = 0$, what is the value of k ?
5. Find the condition that one root of the equation $ax^2 + bx + c = 0$ may be double of other.

LONG ANSWER TYPE

1. The height of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, form the quadratic equation to find the base of the triangle.
2. Determine the positive values of k for which the equation $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ with both have real roots.
3. Find the number of real roots $3^{2x^2-7x+7} = 9$.
4. Solve the following quadratic equation by factorization method $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$, $a + b \neq 0$.
5. If α, β are the roots of equation $x^2 + px + q = 0$, then find out the quadratic equation whose roots are $1 + \frac{\alpha}{\beta}$, $1 + \frac{\beta}{\alpha}$.

TRUE / FALSE TYPE

1. Every quadratic equation has exactly one root.
2. Every quadratic equation has at least one real root.
3. Every quadratic equation has at least two roots.
4. A quadratic equation has exactly two roots, no more no less.
5. If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.

FILL IN THE BLANKS

1. If $b^2 - 4ac = 0$, roots of quadratic equations are real and .....
2. If α and β are zeros of $ax^2 + bx + c = 0$ then $\alpha + \beta = -b$ /.....
3. $b^2 - 4ac > 0$, roots of quadratic equations are real and .....
4. A quadratic equations cannot have more then ..... roots.
5. For $10x^2 + 10x + 1 = 0$ then $\alpha \times \beta$ .....

ANALYTICAL PROBLEM

1. Find the least positive value of k for which the equation $x^2 + kx + 4 = 0$ has real roots.
2. A rectangular form 60 m long and 40 m wide has two concrete cross roads running in the middle of the park and rest of the park has been used as a lawn. If the area of the lawn is 2109 square m, then what is the width of the road ?
3. If α, β are roots of the equation $x^2 + \sqrt{\alpha}x + \beta = 0$, then $\alpha^2 + \beta^2 =$
4. If x = 2 is a root of $3x^2 - 2kx + 2m = 0$ where m = 3, then value of 4k is :
5. If the roots of $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then kb = a + c. Find k.

NUMERICAL PROBLEMS

1. Non-negative root of $\left(\frac{3x-1}{2x+3}\right)^2 - 5\left(\frac{3x-1}{2x+3}\right) + 4 = 0$ is :
2. If $\alpha = \frac{-b+\sqrt{b^2-12c}}{k}$ and $\beta = \frac{-b-\sqrt{b^2-12c}}{k}$ be two roots of the quadratic equation $3x^2 + bx + c = 0$, then value of 3k is :
3. What is the sum of roots of quadratic equation $4x^2 - 12x + 5 = 0$?
4. The difference of squares of two natural numbers is 45. The square of the smaller number is four times the larger number. The sum of numbers is :
5. The speed of a boat in still water is 8 km/hr. It can go 15 km upstream and 22 km downstream in 5 hours. The speed of the stream is :

Answer Key

EXERCISE–I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	B	B	A	D	A	B	B	A	D	B	C	D	C	A
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
C	C	A	A	C	B	C	D	B	C	A	B	C	D	B
31	32	33												
C	B	A												

EXERCISE II

VERY SHORT ANSWER TYPE

1. Real and equal **2.** $x^2 - 2\sqrt{3}x + 2 = 0$ **3.** $2x^2 + x - 6 = 0$ **4.** Two equal real roots

5. $k = 2 \pm 2\sqrt{2}$ **6.** No real roots **7.** – 4 and – 3 **8.** k = 16 **9.** $\frac{-8}{5} < k < \frac{8}{5}$

10. 21, 24, or –21, –24

SHORT ANSWER TYPE

1. $x^2 - 4x - 1 = 0$ **2.** k = 0, 3 **3.** $x = \frac{7}{2}, \frac{3}{4}$ **4.** k = –1 **5.** $2b^2 = 9ac$

LONG ANSWER TYPE

1. $x^2 - 7x - 60 = 0$ **2.** k = 16 **3.** $\frac{5}{2}, 1$ **4.** x = – a or – b **5.** $qx^2 - p^2x + p^2 = 0$

TRUE / FALSE

1. F **2.** F **3.** F **4.** F **5.** T

FILL IN THE BLANKS

1. Equal **2.** a **3.** Unequal **4.** 2 **5.** 10

ANALYTICAL PROBLEM

1. 4 **2.** 3 **3.** 5 **4.** 18 **5.** 2

NUMERICAL PROBLEMS

1. 4 **2.** 18 **3.** 3 **4.** 15 **5.** 3

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : QUDRATICS EQUATIONS)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

ARITHMETIC PROGRESSIONS 5

Concepts

INTRODUCTION

In our earlier classes, we have across various number patterns. Some of them are given as follows :

(i) 2, 5, 8, 11, ... (ii) 20, 16, 12, 8, ... (iii) $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, 4\sqrt{3}$.....

In list (i) successor number is obtained by adding 3 to the previous number.

In list (ii) successor number is obtained by subtracting 4 from the previous number.

In list (iii) successor number is obtained by adding $\sqrt{3}$ to the previous number.

Such patterns are also called sequence. In each of the above sequence the next number can be obtained by some particular rule. So a sequence is a succession of numbers or terms formed according to some rule. The numbers occurring in the sequence are called its terms. The terms of a sequence can be denoted by a_1, a_2, a_3, ...etc, where a_1 is called first term, a_2 second term, a_3 third term and so on. The nth term of a sequence is denoted by a_n.

For sequence, 1, 4, 9, 16, 25 .....

$a_1 = 1 = 1^2$, $a_2 = 4 = 2^2$, $a_3 = 9 = 3^2$, $a_4 = 16 = 4^2$ and so on. The nth term is n^2.

Example 1

Write first three terms of a sequence where n^{th} term is defined by $a_n = n^2 + n$.

Solution :

We have $a_n = n^2 + n$.

Putting n = 1, 2 and 3, we get

$a_1 = 1^2 + 1 = 1 + 1 = 2$

$a_2 = 2^2 + 2 = 4 + 2 = 6$

$a_3 = 3^2 + 3 = 9 + 3 = 12$

Hence first three terms are 2, 6 and 12

1. ARITHMETIC PROGRESSION

A sequence is called an arithmetic progression if the difference of a term and its predecessor is always constant. This constant is called the common difference of the arithmetic progression. So common difference $d = a_n - a_{n-1}$.

Thus if a is first term and d is common difference, then the successive terms of an A.P. are a, a + d, a + 2d, a + 3d, ... It is also called the general form of an A.P.

Note : d can be negative, positive or zero.

Focus Point

The numbers in an A.P. will remain in A.P. if any constant is added, subtracted, multiplied or divided.

1.1 THERE ARE TWO TYPES OF A.P.

(i) Finite A.P.

(ii) Infinite A.P.

(i) An A.P. in which number of terms are finite, is called finite A.P.

Examples : 10, 15, 20, 20, 30, 35, 40.

17, 20, 23, ..., 47.

In both A.Ps. number of terms are 7 and 11 which is finite.

(ii) An A.P. in which numbers of terms are infinite is called infinite A.P.

Example : 1, 2, 3, 4, 5, ...

2, 4, 6, 8, 10, ...

Both A. Ps. are of natural numbers and even numbers respectively and there are infinite number of terms.

Example 2

Identify which of the following sequence an A.P.

(i) 1.2, 3.2, 5.2, 7.2, ...

(ii) 5, 10, 15, 20, ...

(iii) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$.....

(iv) 5, 5, 5, 5, 5, ...

Solution :

(i) Here $a_1 = 1.2$, $a_2 = 3.2$, $a_3 = 5.2$ and $a_4 = 7.2$

Now $a_2 - a_1 = 3.2 - 1.2 = 2$; $a_3 - a_2 = 5.2 - 3.2 = 2$

$a_4 - a_3 = 7.2 - 5.2 = 2$

Since common difference is a constant, so given sequence is an A.P.

(ii) Here $a_1 = 5$, $a_2 = 10$, $a_3 = 15$ and $a_4 = 20$

$a_2 - a_1 = 10 - 5 = 5$; $a_3 - a_2 = 15 - 10 = 5$

$a_4 - a_3 = 20 - 15 = 5$

$\Rightarrow$ d = 5 (constant)

Since common difference is constant.

$\therefore$ It is an A.P.

(iii) Here $a_1 = \frac{1}{2}, a_2 = \frac{2}{3}, a_3 = \frac{3}{4}, a_4 = \frac{4}{5}$

Now $a_2 - a_1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

$a_3 - a_2 = \frac{3}{4} - \frac{2}{3} = \frac{1}{12}$; $a_4 - a_3 = \frac{4}{5} - \frac{3}{4} = \frac{1}{20}$

Since common difference is not constant

$\therefore$ Given sequence is not an A.P.

(iv) Here $a_1 = 5$, $a_2 = 5$, $a_3 = 5$, $a_4 = 5$

Now $a_2 - a_1 = 5 - 5 = 0$, $a_3 - a_2 = 5 - 5 = 0$

$a_4 - a_3 = 5 - 5 = 0$

Since common difference is constant given sequence is an A.P.

Note : Such A.P. is called constant A.P.

Example 3

Write first three terms of the A.P. where first term and common difference is 7 and 9 respectively.

Solution :

We have a = 7 and d = 9

So first term a = 7

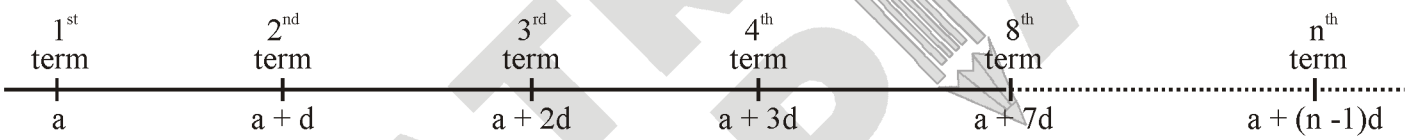
Second term a + d = 7 + 9 = 19

Third term = a + 2d = 7 + 2 × 9 = 7 + 18 = 25

So first three terms are 7, 16 and 25.

2. GENERAL TERM OF AN A.P. (n^{th} TERM)

Let us consider the A.P. a, a + d, a + 2d..... . It has first term = a and common difference = d.



$a_2 = a + (2 - 1)d$, $a_3 = a + (3 - 1)d$, $a_4 = a + (4 - 1)d$ $a_8 = a + (8 - 1)d$

The general terms of an A.P. is also called n^{th} term of the A.P. and is given by $a_n = a + (n - 1)d$

Note : (i) An A.P. is a linear function in which the common difference is independent of first term and value of n.

(ii) The last term is often deonted by l and given by $l = a + (n - 1)d$.

3. n^{th} TERM FROM END OF AN A.P.

If a and d be respectively the first term and common difference of an A.P. and suppose there are m terms in the A.P., then n^{th} term (n < m) from the end = $(m - n + 1)^{th}$ term from beginning. So, n^{th} term from end = a + (m – n)d.

ALTERNATIVELY

We can take the last term (if it is given) as first term and negative of the common difference to form the reserve A.P. of the given A.P. and find the nth term.

$\therefore$ n^{th} term from end = last term + (n – 1)(–d) = l – (n – 1)d.

Example 4

Find how many terms are there in the A.P. 16, 24, 32, ... 96.

Solution :

Here a = 16, d = 24 – 16 = 8 and last term = 96.

Let 96 be n^{th} term of the A.P., So $a_n = 96$

$\Rightarrow$ $a + (n - 1)d = 96$ $\Rightarrow$ $16 + (n - 1)8 = 96$

$\Rightarrow$ $(n - 1)8 = 80$ $\Rightarrow$ $(n - 1) = 10$ $\Rightarrow$ $n = 11$

So there are 11 terms in the A.P.

Example 5

How many 4-digit numbers are there which is divisible by 21.

Solution :

The smallest 4 digit number divisible by 21 is 1008 and the largest is 9996. So we get an A.P. as 1008, 1029, 1050, ...,9996.

Here a = 1008, d = 21. Let 9996 be n^{th} term.

So $a + (n - 1)d = 9996$ $\Rightarrow$ $1008 + 21(n - 1) = 9996$

$\Rightarrow$ $21(n - 1) = 8988$ $\Rightarrow$ $(n - 1) = 428$ $\Rightarrow$ $n = 429$

Hence, there are 429 such numbers.

Example 6

If the nth term of an A.P. is $(5n - 2)$, find its.

(i) first term

(ii) common difference and

(iii) 19th term.

Solution :

$a_n = (5n - 2)$ (Given)

$\Rightarrow$ $a_1 = (5 \times 1 - 2) = 3$ and $a_2 = (5 \times 2 - 2) = 8$

Thus, we have

(i) First term = 3

(ii) Common difference = $a_2 - a_1 = (8 - 3) = 5$

(iii) 19th term = $a + (19 - 1)d$,

where, a = 3 and d = 5 so, $a_{19} = (3 + 18 \times 5) = 93$.

Example 7

The 6th term of an A.P. is –10 and its 10th term is –26. Determine the 15th term of the A.P.

Solution :

$a_n = a + (n - 1)d$

$\Rightarrow$ $a_6 = a + (6 - 1)d$ and $a_{10} = a + (10 - 1)d$

$\Rightarrow$ $a_6 = a + 5d$ and $a_{10} = a + 9d$

$\therefore$ $a + 5d = -10$ .....(1)

and $a + 9d = -26$ .....(2)

On subtracting (i) from (ii), we get

$4d = -16 \Rightarrow d = -4$

On subtracting $d = -4$ in (i), we get

$a + 5 \times (-4) = -10 \Rightarrow a = 10$

Thus, $a = 10$ and $d = -4$

$\therefore$ 15th term $= a_{15} = a + (15 - 1)d$

$= (a + 14d) = [10 + 14 \times (-4)] = (10 - 56) = -46$

Example 8

Find the 10th term from the end of the A.P. 4, 9, 14, ..., 254.

Solution :

Here $a = 4$, $d = (9 - 4) = 5$, $l = 254$ and $n = 10$

Now, n^{th} term from the end $= \{l - (n - 1)d\}$

$\therefore$ 10th term from the end $\{254 - (10 - 1)5\} = \{254 - (9 \times 5)\} = (254 - 45) = 209$

Hence, the 10th term from the end is 209.

Focus Point

(i) If a, b, c are in A.P., then $b - a = c - b$ = common difference $\Rightarrow 2b = a + c$

Thus a, b, c are in A.P. iff $2b = a + c$. Here b is called arithmetic mean (A.M.) between a and c.

(ii) Arithmetic mean between a and b is $\frac{a+b}{2}$.

4. SELECTION OF TERMS OF AN A.P.

The convenient choice for selection of some terms in an A.P. are in the table.

Number of terms	Terms	Common difference
3	$a - d, a, a + d$	d
4	$a - 3d, a - d, a + d, a + 3d$	2d
5	$a - 2d, a - d, a, a + d, a + 2d$	d
6	$a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$	2d
7	$a - 3d, a - 2d, a - d, a, a + d, a + 2d, a + 3d$	d

Example 9

The sum of three numbers in A.P. is – 3 and their product is 8. Find the numbers.

Solution :

Let the numbers be (a – d), a, (a + d). Then,

sum = – 3 $\Rightarrow$ (a – d) + a + (a + d) = – 3 $\Rightarrow$ 3a = – 3 $\Rightarrow$ a = – 1

Now, product = 8

$\Rightarrow$ (a – d)(a) (a + d) = 8 $\Rightarrow$ $a(a^2 - d^2) = 8$

$\Rightarrow$ $(-1)(1 - d^2) = 8$ $\Rightarrow$ $d^2 = 9$ $\Rightarrow$ $d = \pm 3$

If d = 3, the numbers are –4, –1, 2.

If d = – 3, the numbers are 2, –1, –4

Thus, the numbers are – 4, –1, 2 or 2, –1, – 4.

Example 10

Find the arithmetic mean between

(i) 13 and 19

(ii) (a – b) and (a + b)

Solution :

(i) A.M. between 13 and 19 $= \frac{1}{2}(13 + 19) = 16$

(ii) A.M. between (a – b) and (a + b) $= \frac{1}{2}[(a-b)+(a+b)] = a$

5. SUM OF FIRST N TERMS OF AN A.P.

If $a_1, a_2, a_3, \ldots, a_n$ is a sequence, then the expansion $a_1 + a_2 + a_3 + \ldots + a_n$ is called the series corresponding to the sequence. Thus, the series corresponding to A.P. a, a + d, a + 2d, ..., a + (n – 1)d is given by (a) + (a + d) + (a + 2d) + ... + [a + (n – 1)d].

5.1 FORMULA FOR THE SUM OF N-TERMS OF AN A.P.

Let S_n represents the sum upto n terms of the given arithmetic progression (or series).

$\Rightarrow$ $S_n = \frac{n}{2}[2a + (n-1)d]$, where a is first term and d is common difference.

Proof : Let S_n denote the sum of first n terms of an A.P., with first term 'a' and common difference 'd'.

Thus, $S_n = a + (a + d) + (a + 2d) + \ldots + (a + (n-2)d) + (a + (n-1)d)$ .....(1)

We can also write it in reverse order as

$\Rightarrow$ $S_n = (a + (n-1)d) + (a + (n-2)d) + (a + (n-3)d) + \ldots + (a + d) + a$ .....(2)

Adding (1) and (2), we get,

$\Rightarrow \quad 2S_n = [2a + (n-1)d] + [2a + (n-1)d] + [2a + (n-1)d] + ... + [2a + (n-1)d] + [2a + (n-1)d]$

$\Rightarrow \quad 2S_n = n[2a + (n-1)d]$

$\Rightarrow \quad S_n = \frac{n}{2}[2a + (n-1)d]$

Focus Point

(i) In case of last term of A.P. is given, the above formula can be written as

$S_n = \frac{n}{2}[a + a + (n-1)d] \qquad \Rightarrow \quad S_n = \frac{n}{2}[a + l]$, where $l = a + (n-1)d$ is last term.

(ii) From sum of n terms S_n, n^{th} term of the A.P. can be obtained as $a_n = S_n - S_{n-1}$.

(iii) The sum of first n natural numbers is given by $\frac{n(n+1)}{2}$

(iv) The sum of square of first n natural numbers is given by $\frac{n(n+1)(2n+1)}{6}$

(v) The sum of cube of first n natural numbers is given by $\left[\frac{n(n+1)}{2}\right]^2$

Example 11

Find the sum of the first 20 terms of the A.P. : 5, 8, 11, 14, ...

Solution :

First term of the A.P. = 5, *i.e.*, a = 5

Common difference = 3, *i.e.*, d = 3

$\therefore \quad S_{20} = \frac{20}{2}\{2a + 19d\} = 10 \times \{10 + 19 \times 3\} = 10 \times 67 = 670.$

Example 12

If S_1, S_2, S_3 be the sum of n, 2n and 3n terms respectively of an A.P., prove that $S_3 = 3(S_2 - S_1)$.

Solution :

Let a be the 1st term and d be the common difference of an A.P.

S_1 = Sum of n terms of the A.P. $= \frac{n}{2}[2a + (n-1)d]$

S_2 = Sum of 2n terms of the A.P. $= \frac{2n}{2}[2a + (2n-1)d] = n[2a + (2n-1)d]$

S_3 = Sum of 3n terms of the A.P. $= \frac{3n}{2}[2a + (3n-1)d]$

Now, R.H.S. $= 3(S_2 - S_1) = 3\left[n\{2a + (2n-1)d\} - \frac{n}{2}\{2a + (n-1)d\}\right]$

$= 3 \times \frac{n}{2}[\{4a + 2(2n-1)d\} - \{2a + (n-1)d\}]$

$= \frac{3n}{2}[4a + (4n-2)d - 2a - (n-1)d]$

$= \frac{3n}{2}[2a + (3n-1)d] = S_3 =$ L.H.S.

Example 13

If the sum of n terms of an A.P. is given by $S_n = (3n^2 + 2n)$, find its

(i) n^{th} term

(ii) first term

(iii) common difference.

Solution :

$S_n = (3n^2 + 2n)$

$S_{n-1} = 3(n-1)^2 + 2(n-1)$

$\therefore \quad S_{n-1} = \{3n^2 - 4n + 1\}$

(i) The nth term is given by

$T_n = (S_n - S_{n-1}) = \{(3n^2 + 2n) - (3n^2 - 4n + 1)\} = (6n - 1)$

$\therefore \quad n^{th}$ term $= (6n - 1)$ .....(1)

(ii) Putting n = 1 in (1), we get

$T_1 = (6 \times 1 - 1) = 5$

$\therefore$ First term = 5

(iii) Putting n = 2 in (1), we get

$T_2 = (6 \times 2 - 1) = 11$

$\therefore \quad d = (T_2 - T_1) = (11 - 5) = 6$

SOLVED EXAMPLES

SE. 1

Show that the progression 11, 6, 1, – 4, –9, ... is an A.P. Find its first term and the common difference.

Ans. Clearly, (6 – 11) = (1 – 6) = (– 4 – 1) = (– 9 + 4) = – 5 (constant)

Thus, each term differs from its preceding term by – 5. So, the given progression is an A.P.

Its first term = 11 and common difference = –5.

SE. 2

What is 18th term of the sequence defined by $a_n = \frac{n(n-3)}{(n+4)}$

Ans. We have, $a_n = \frac{n(n-3)}{n+4}$

Putting n = 18, we get

$$a_{18} = \frac{18\times(18-3)}{18+4} = \frac{18\times15}{22} = \frac{135}{11}$$

SE. 3

Find the sum of first 24 terms of the list of numbers whose n^{th} term is given by $a_n = 3 + 2n$.

Ans. As $a_n = 3 + 2n$

So, $a_1 = 3 + 2 = 5$

$a_2 = 3 + 2 \times 2 = 7$

$a_3 = 3 + 2 \times 3 = 9$

.....

.....

List of numbers becomes 5, 7, 9, 11, ... Here 7 – 5 = 9 – 7 = 11 – 9 = 2 and so on. So it forms and A.P. with common difference d = 2.

To find S_{24}, we have n = 24, a = 5, d = 2

Therefore,

$$S_{24} = \frac{24}{2}[2\times5+(24-1)\times2] = 12[10+46] = 672$$

So, sum of first 24 terms of the list of numbers is 672.

SE. 4

If the pth, q^{th} and n^{th} terms of an A.P. be a, b, c respectively, then slow that.

a(q – r) + b(r – p) + c (p – q) = 0

Ans. Let x be the first term and d be the common difference of the given A.P. Then

$t_p = x + (p-1)d$, $t_q = x + (q-1)d$ and $t_r = x + (r-1)d$

$\therefore$ $x + (p-1)d = a$ .....(1)

$x + (q-1)d = b$ .....(2)

$x + (r-1)d = c$ .....(3)

On multiplying (1) by (q – r), (2) by (r – p) and (3) by (p – q) and adding, we get

a(q – r) + b(r – p) + c(p – q) = x (q – r + r – p + p – q) + d[(p – 1) (q – r) + (q – 1) (r – p) + (r – 1) (p – q)] = x × 0 + d × 0

Hence, a(q – r) + b(r – p) + c(p – q) = 0.

SE. 5

If a, b, c are in A.P., show that

(i) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in A.P.

(ii) $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in A.P.

Ans. (i) Given a, b, c are in A.P.

$\Rightarrow$ $b - a = c - b$ .....(1)

Now, if $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in A.P.,

then $\frac{1}{ca} - \frac{1}{bc} = \frac{1}{ab} - \frac{1}{ca}$

$\Rightarrow$ $\frac{b-a}{abc} = \frac{c-b}{abc}$

$\Rightarrow$ $b - a = c - b$

This is true from (1)

Hence, $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in A.P.

(ii) Let $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in A.P.

$\Rightarrow$ $a^2(b + c) + abc$, $b^2(c + a) + abc$, $c^2(a + b) + abc$ are in A.P. [Adding abc to each term]

$\Rightarrow$ $a(ab + ac + bc)$, $b(bc + ab + ac)$, $c(ca + cb + ab)$ are in A.P.

$\Rightarrow$ a, b, c are in A.P.

[Dividing each term by $(ab + bc + ca)$]

This is given to be true.

$\therefore$ $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in A.P.

SE. 6

If the nth term of the A.P. 9, 7, 5, ... is same as the nth term of the A.P. 15, 12, 9, ... find n.

Ans. Given A.P.s are 9, 7, 5 .... and 15, 12, 9 .....

Let a_1, d_1 and a_2, d_2 be the first terms and common difference of two A.P.s respectively. So

$a_1 = 9$, $d_1 = -2$, $a_2 = 15$, $d_2 = -3$.

According to question n^{th} term of two A.P.s are same

$\Rightarrow$ $a_1 + (n - 1)d_1 = a_2 + (n - 1)d_2$

$\Rightarrow$ $9 + (n - 1)(-2) = 15 + (n - 1)(-3)$

$\Rightarrow$ $3(n - 1) - 2(n - 1) = 15 - 9$

$\Rightarrow$ $(n - 1) = 6$

$\Rightarrow$ $n = 7$

SE. 7

The sum of three numbers in A.P. is 12 and the sum of their cubes is 288. Find the numbers.

Ans. Let three numbers in A.P. be $a - d$, a, $a + d$

So, $a - d + a + a + d = 12$

$\Rightarrow$ $3a = 12$ $\Rightarrow$ $a = 4$

Also $(a - d)^3 + a^3 + (a + d)^3 = 288$

$\Rightarrow$ $(4 - d)^3 + 4^3 + (4 + d)^3 = 288$

$\Rightarrow$ $64 - d^3 - 48d + 12d^2 + 64 + 64 + d^3 + 48d + 12d^2 = 288$

$\Rightarrow$ $192 + 24d^2 = 288$

$\Rightarrow$ $24d^2 = 96$ $\Rightarrow$ $d^2 = 4$

$\therefore$ $d = \pm 2$

So numbers are

$4 - 2, 4, 4 + 2$ or $4-(-2), 4, 4 + (-2)$

i.e., 2, 4, 6 or 6, 4, 2

SE. 8

Find the value of x for which $(8x + 4)$, $(6x - 2)$ and $(2x + 7)$ are in A.P.

Ans. $(8x + 4)$, $(6x - 2)$ and $2x + 7$ are in A.P. iff

$8x + 4 + 2x + 7 = 2(6x - 2)$

$\Rightarrow$ $10x + 11 = 12x - 4$

$\Rightarrow$ $12x - 10x = 11 + 4$

$\Rightarrow$ $2x = 15$

$\Rightarrow$ $x = \frac{15}{2}$

SE. 9

Find the sum of all integers between 50 and 500, which are divisible by 7.

The smallest and largest numbers between 50 and 500 divisible by 7 are 56 and 497 respectively. So we get 56, 63, 70 .... 497 as an A.P. having first term 56 and common difference 7. Now let 497 be n^{th} term of this A.P.

So $a_n = 497$

$\Rightarrow$ $56 + (n - 1) \times 7 = 497$

$\Rightarrow$ $7(n - 1) = 441$

$\Rightarrow$ $(n - 1) = 63$

$\Rightarrow$ $n = 64$

Now required sum

$$= \frac{n}{2}(a+l) = \frac{64}{2}(56+497) = \frac{64\times 553}{2} = 17696$$

SE. 10

If the sum of 7 terms of an A.P. is 49 and that of 17 terms is 289, find the sum of n terms.

Let a and d be respectively first term and common difference of given A.P. so we have

$$S_7 = 49 \Rightarrow \frac{7}{2}(2a+6d) = 49$$

$\Rightarrow$ $2a + 6d = 14$ .....(i)

and $S_{17} = 289$

$\Rightarrow$ $\frac{17}{2}(2a + 16d) = 289$

$\Rightarrow$ $2a + 16d = 34$ .....(ii)

Subtracting (i) from (ii) we get

$10d = 20$ $\Rightarrow$ $d = 2$ and by (i) $a = 1$

So sum of first n terms is

$$S_n = \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[2.1+(n-1)2]$$

$$= \frac{n}{2}[2+2n-2] = \frac{n\times 2n}{2} = n^2$$

SE. 11

In an A.P. the first term is 22, n^{th} term is –11 and the sum to first n terms is 66. Find n and d, the common difference.

Ans. We have $a = 22$, $a_n - 11$ and $S_n = 66$

Now $a_n = -11$

$\Rightarrow$ $a + (n - 1)d = -11$

$\Rightarrow$ $(n - 1)d = -33$ .....(i)

and $S_n = 66$ $\Rightarrow$ $= \frac{n}{2}[2a+(n-1)d] = 66$

$\Rightarrow$ $n[2 \times 22 + (n - 1)d] = 132$

$\Rightarrow$ $n[44 - 33] = 132$ [Using ...(i)]

$\Rightarrow$ $11n = 132$

$\Rightarrow$ $n = 12$

By (i) $11d = -33$ $\Rightarrow$ $d = -3$

So, number of terms = 12 and common difference = –3

EXERCISE – I

ONLY ONE CORRECT TYPE

1. Which of the following is a term of the sequence 3, 7, 11, ...?
(A) 184 (B) 185
(C) 186 (D) 187

2. How many terms are there in the sequence: 3, 17, 31, ..., 101 ?
(A) 7 (B) 6
(C) 8 (D) 9

3. An A.P. has general term 2n + 5. If any term of the A.P. is divided by 2, then the remainder is.
(A) 5 (B) 3
(C) 1 (D) No fixed

4. Sum of first n terms of an A.P. is 0 one of the terms of the A.P., is also 0 if.
(A) n is odd and greater than 1
(B) n is multiple of 4
(C) n is even
(D) None of these

5. If 9th term from the end of an A.P. is 12th term from the beginning, then the number of terms is.
(A) 21 (B) 20
(C) 19 (D) 22

6. If the general term of an A.P. be known, then we can determine.
(A) its common difference only
(B) any term of it
(C) the number of terms
(D) the first term only

7. If the first and 10th terms of an A.P. be known, then the common difference of the A.P. can be determined by dividing the difference of the tenth and first terms by.
(A) 10 (B) 11
(C) 9 (D) None of these

8. There is an A.P. consisting of 20 terms. The general term is given by 20n + 5, then its common difference is.
(A) 5 (B) 25
(C) 20 (D) 15

9. The numbers x, $\frac{1}{2}$, $\frac{1}{3}$ are in A.P. then x is equal to.
(A) 1 (B) $\frac{2}{3}$
(C) $\frac{3}{2}$ (D) None of these

10. If 7th and 13th terms of an A.P. be 34 and 64 respectively, then its 18th term is.
(A) 87 (B) 88
(C) 89 (D) 90

11. If n^{th} terms of the sequences 3, 10, 17,.....and 63, 65, 67,..... are equal the value of n is.
(A) 12 (B) 13
(C) 14 (D) 15

12. The number of terms in 5, 8, 11, 14,.....95 is.
(A) 31 (B) 32
(C) 33 (D) 34

13. The sum of all 2 digit odd numbers is.
(A) 2475 (B) 2530
(C) 4905 (D) 5049

14. If the sum of sequence 2,5, 8, 11.....is 60100, the number of terms is.
(A) 100 (B) 200
(C) 150 (D) 250

15. The second term of an A.P. (x –y) and 5th term is (x + y), then its first term is.
(A) $x - \frac{y}{3}$ (B) $x - \frac{2}{3}y$
(C) $x - \frac{4}{3}y$ (D) $x - \frac{5}{3}y$

16. If the pth term of an A.P. is q and the qth term is p, then r^{th} term is.
(A) q – p + r (B) p – q + r
(C) p + q + r (D) p + q – r

17. If $S_n = nP + \frac{n(n-1)}{2}Q$, where S_n denotes the sum of the first n terms of an A.P., the common difference is.
(A) P + Q (B) 2P + 3Q
(C) 2Q (D) Q

18. The sum of integers from 1 to 100 that are divisible by 2 or 5 is.
(A) 1525 (B) 3050
(C) 3000 (D) None of these

19. The sums of n terms of two AP series are in the ratio of (n + 1) : (n + 3). Then ratio of their sixth terms is.
(A) $\frac{3}{7}$ (B) $\frac{7}{3}$
(C) $\frac{6}{7}$ (D) $\frac{7}{6}$

20. If the sum of n terms of an A.P. series is n^2, then the common difference is.
(A) 2 (B) 3
(C) 4 (D) 5

21. The sum of 19 terms of an A.P. whose n^{th} term is 2n + 1 is.
(A) 398 (B) 399
(C) 698 (D) 42

22. The sum of n terms of 2 arithmetic progression are in the ratio of (7n + 1) : (4n + 27). The ratio of their 11th terms is.
(A) 162 : 119 (B) 111 : 148
(C) 121 : 148 (D) 148 : 111

23. If the first term of an A.P. is 2 and the sum of first 5 terms is equal to one fourth of the sum of the next five terms, then the sum of first 30 terms is.
(A) 2550 (B) –2550
(C) 5100 (D) –5100

24. If x, y, z are in A.P. then the value of (x + y – z) (y + z –x) is.
(A) $8yz - 3y^2 - 4z^2$ (B) $4xz - 3y^2$
(C) $8xy - 4x^2 - 3y^2$ (D) $10xz - 3x^2 - 3z^2$

25. If the sum of 3 consective terms of an inscreasing AP is 51, and the product of the first and third of these terms is 273, then the third term is.
(A) 13 (B) 9
(C) 21 (D) 17

PARAGRAPH TYPE

PASSAGE # I

The sum of first n terms of an A.P. is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

26. Find the sum of first 15 multiples of 8.
(A) 840 (B) 1020
(C) 960 (D) 920

27. Find the sum of the first 51 terms of the A.P. whose 2nd term is 2 and 4th term is 8.
(A) 4170 (B) 2970
(C) 3720 (D) 3774

28. Find the sum of first 10 terms of the A.P. x – 8, x – 2, x + 4,.....
(A) 10x + 210 (B) 10x + 190
(C) 5x + 190 (D) 5x + 210

PASSAGE # II

If S_n is the sum of n terms of an A.P., then the nth term t_n of the sequence can be determined by $t_n = S_n - S_{n-1}$.

29. If the sum of n terms of an A.P. is given by $S_n = (3n^2 + 2n)$, find its nth term.

(A) 6n + 1 (B) 6n – 1

(C) 4n – 2 (D) 4n – 1

30. If sum of n terms of an A.P. is $4n^2 + 7n$, find the 15th term.

(A) 123 (B) 142

(C) 153 (D) 136

31. If sum of n terms of an A.P. is $n^2 + 13n$, find its 13th term.

(A) 24 (B) 12

(C) 38 (D) 25

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from **List – I** and **List – II** are given as options (a), (b), (c) and (d) out of which one is correct.

32. Match the A.P. given in List – I with their common difference given in List – II.

List – I	**List – II**
(P) $1, \frac{3}{2}, 2, \frac{5}{2}, ...$	(i) – 4
(Q) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, ...$	(ii) 0.2
(R) 1.8, 2.0, 2.2, 2.4	(iii) $\frac{4}{3}$
(S) 0, – 4, – 8, – 12	(iv) $\frac{1}{2}$

(A) (P) → (i), (Q) → (iv), (R) → (iii), (S) → (ii)

(B) (P) → (i), (Q) → (ii), (R) → (iv), (S) → (iii)

(C) (P) → (iv), (Q) → (iii), (R) → (ii), (S) → (i)

(D) (P) → (iv), (Q) → (ii), (R) → (i), (S) → (iii)

33. Match the List – I with List – II.

List – I	**List – II**
(P) Sum of the first 20 terms of A.P. – 6, 0, 6, 12,.....is	(i) 7500
(Q) Sum of the first 14 terms of an A.P. is 1050 and its first term is 10. Its 20th term is	(ii) 1020
(R) Sum of the A.P., 1 + 3 + 5 + ...+ 199 is	(iii) 200
(S) Sum of all odd numbers between 100 and 200 is	(iv) 10000

(A) (P) → (ii), (Q) → (iv), (R) → (iii), (S) → (i)

(B) (P) → (ii), (Q) → (iii), (R) → (iv), (S) → (i)

(C) (P) → (iv), (Q) → (iii), (R) → (i), (S) → (ii)

(D) (P) → (iv), (Q) → (ii), (R) → (i), (S) → (iii)

EXERCISE – II

VERY SHORT ANSWER TYPE

1. Let a, b, c be in A.P., then prov that 2a + 3, 2b + 3, 2c + 3 are also in A.P.
2. Find the number of terms in the A.P. $-1, -\frac{5}{6}, -\frac{2}{3}, -\frac{1}{2}, \ldots\ldots, \frac{10}{3}$.
3. If x + 2, 2x + 5 and 4x + 8 are in A.P., then find the value of x.
4. Find the 15th term of an A.P., whose general term is 10– 5n.
5. Find the common difference of an A.P. whose general term is 3n + 5.
6. Find the 10th term from the end of the A.P. 1, 4, 7, 10.....91.
7. Find the first terms of an A.P., whose 3rd term is 3 and 8th term is 33.
8. Show that the sequence defined by $a_n = 5n^2 + 5$ is not an A.P.
9. Find the first three terms of an A.P. whose nth term is 5n + 3.
10. How many two digits numbers are divisible by 3.

SHORT ANSWER TYPE

1. For what value of n is the nth term of the following two A.P.'s will be same ?
   (i) 1,7, 13, 19 (ii) 69, 68, 67,.....
2. Which term of the sequence 4, 9, 14, 19.....is 124.
3. The angles of a quadrlilateral are in A.P. whose common difference is 10°. Find the angles.
4. Find the sum of all odd numbers between 100 and 200.
5. Find the sum of first 15 terms of the sequence whose nth term is 3 + 4n.

LONG ANSWER TYPE

1. If the first and third terms of an A.P. are $(a-b)^3$ and $(a+b)^3$, find the second term.
2. Find the value of x for which (8x + 4), (6x –2) and (2x + 7) are in A.P.
3. x_1, x_2, x_3 ..... are in A.P. If $x_1 + x_7 + x_{10} = -6$ and $x_3 + x_8 + x_{12} = -11$, then find value of $x_3 + x_8 + x_{22}$.
4. If a,b,c are in A.P., then show that $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are also in A.P.
5. The sum of three consecutive terms of an A.P. is 21 and the sum of the squares of these terms is 165. The middle term of the three terms is.

TRUE/FALSE

1. An AP is a list of numbers in which each term is obtained by adding a fixed number to the preceding number.
2. The series is identified as an AP with the help of a common ratio between consecutive terms.
3. Sum of first n terms, $S_n = \frac{n}{2}[a - l]$.
4. The nth term, $T_n = a + (n-1)d$.
5. Common difference in AP is denoted by "d".

FILL IN THE BLANKS

1. The sum of first n natural numbers is _____ .
2. After multiplying each term of the AP with a fixed number, the _____ is multiplied by the same fixed number.
3. If a, b, c are in AP then common difference is ____ .
4. An AP is a sequence where the differences between every two _____ terms are the same.
5. An AP is a sequence where each term, except the first term, is obtained by adding a fixed number to its _____ .

ANALYTICAL TYPE

1. The sum of the third and seventh terms of an A.P. is 6 and their product is 8, then common difference –

   (A) ± 1 (B) ± 2

   (C) $\pm\frac{1}{2}$ (D) $\pm\frac{1}{4}$

2. The sum of all two digit numbers each of which leaves remainder 3 when divided by 5 is :

   (A) 952 (B) 999

   (C) 1064 (D) 1120

3. If a_1, a_2....., a_{19} are the first 19 terms of an A.P. and $a_1 + a_8 + a_{12} + a_{19} = 224$. Then $\sum_{i=1}^{19} a_i$ is equal to :

   (A) 896 (B) 1064

   (C) 1120 (D) 1164

4. The sum of 18 consecutive natural numbers is a perfect square. The smallest possible value of this sum is :

   (A) 144 (B) 169

   (C) 225 (D) 289

5. If a_1, a_2, a_3,..... is an arithmetic progression with common difference 1 and $\sum_{i=1}^{98} a_i = 137$, then the value of $a_2 + a_4 + a_6 + \ldots\ldots\ldots + a_{98}$ is :

   (A) 67 (B) 83

   (C) 93 (D) 98

NUMERICAL PROBLEMS

1. The sum of n terms of an A.P. is $\left(\frac{5n^2}{2}+\frac{3n}{2}\right)$. If a_{20} be 20th term of the A.P., find a_{20}.
2. If a, 2(a + 5) and 2(4a – 5) are in A.P., find value of a.
3. Sum of three numbers in A.P. is 21 and their product is 231. Find the largest number.
4. If 10 times the 10th term of an A.P. is equal to 15 times the 15th term, then its 25th term is
5. If the sum of first 24 terms of a sequence whose n^{th} term is given by $t_n = 3 + \frac{2n}{3}$ is k, then what is the value of k ?

Answer Key

EXERCISE–I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	C	C	A	B	B	C	C	B	C	B	A	A	B	D
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
D	D	B	C	A	B	D	B	C	C	C	D	B	B	A
31	32	33												
C	C	B												

EXERCISE II

VERY SHORT ANSWER TYPE

2. 27 **3.** 0 **4.** – 65 **5.** 3 **6.** 64 **7.** –9 **9.** 8, 13, 18
10. 30

SHORT ANSWER TYPE

1. No value of n **2.** 25 **3.** 75°, 85°, 95°, 105° **4.** 7500 **5.** 525

LONG ANSWER TYPE

1. $a^3 + 3ab^2$ **2.** $x = \frac{15}{2}$ **3.** – 21 **5 .** 7

TRUE/FALSE

1. T **2.** F **3.** F **4.** T **5.** T

FILL IN THE BLANKS

1. $\frac{n(n+1)}{2}$ **2.** Common difference **3.** $b - a$ **4.** Consecutive
5. Previous term

ANALYTICAL

1. C 2. B 3. B 4. C 5. C

NUMBRICAL

1. 99 2. 6 3. 11 4. 0 5. 272

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : ARITHMETIC PROGRESSION)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

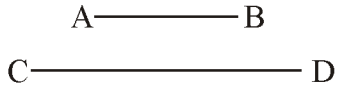
TRIANGLES 6

Concepts

INTRODUCTION

In previous classes, we have learnt about the congruency of two geometric figure. In this chapter we shall learn about these geometric figures which have the same shape but not necessary have the same size. These kind of geometric figures are known as similar figures. So the congruent figures are always similar figures but similar figures need not be congruent figures.

(i) Two line segment are similar and the two line segment are congruent if they have the same length



(ii) Two circles are similar and the two circles are congruent if they have the same radius

1. SIMILAR FIGURES

1.1 SIMILAR POLYGONS

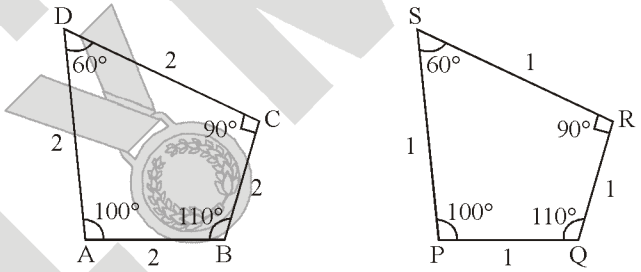
Two polygons of the same number of sides are said to be similar. If

(i) Their corresponding angles are equal

(ii) Their corresponding sides are in the same ratio

Example 1

If two polygons ABCD and PQRS are similar then



Solution :

By the definition

$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R, \angle D = \angle S$

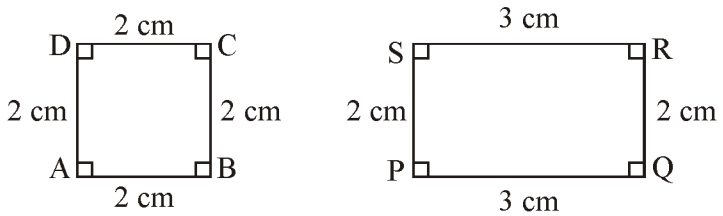
and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{DA}{SP} = 2$

So corresponding sides are proportional.

Therefore quadrilateral ABCD and PQRS are similar

Example 2

State whether the following quadrilaterals are similar or not :



Solution :

Clearly square ABCD and rectangle PQRS are equiangular.

But corresponding sides of square ABCD and rectangle PQRS are not proportional.

Therefore square ABCD and rectangle PQRS are not similar.

2. SIMILARITY OF TRIANGLES

Two triangles are said to be similar if

(i) Their corresponding angles are equal (or triangles are equiangular)

(ii) Their corresponding sides are in the same ratio (or proportional)

For example, $\triangle ABC$ and $\triangle PQR$ are similar if

(i) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ [Correspnding sides are proportional]

(ii) $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ [Corresponding angles are equal]

3. BASIC PROPORTIONALITY THEOREM (BPT) OR THALES THEOREM

3.1 THEOREM 1

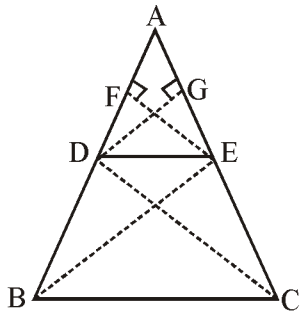
Statement : If a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, the other two sides are divided in the same ratio.

Given : A triangle ABC in which DE || BC and DE intersects AB at D and AC at E.

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join BE, CD and draw EF $\perp$ AB, DG $\perp$ AC.

Proof : In $\triangle EAD$ and $\triangle EDB$, as EF is perpendicular to AB, therefore EF is the height for both triangles EAD and EDB.



Now, area of $\Delta EAD = \frac{1}{2} \times (\text{Base} \times \text{height}) = \frac{1}{2} \times AD \times EF$

Again, area of $\Delta EDB = \frac{1}{2} \times (\text{Base} \times \text{height}) = \frac{1}{2} \times DB \times EF$

$\therefore \frac{\text{Area}(\Delta EAD)}{\text{Area}(\Delta EDB)} = \frac{AD}{DB}$ .....(1)

Similarly, $\frac{\text{Area}(\Delta EAD)}{\text{Area}(\Delta ECD)} = \frac{AE}{EC}$ .....(2)

Since, triangles EDB and ECD are on the same base and between the same parallel lines DE and BC therefore, Area ΔEDB = Area ΔECD.

$\Rightarrow$ From (i) and (ii) $\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$.

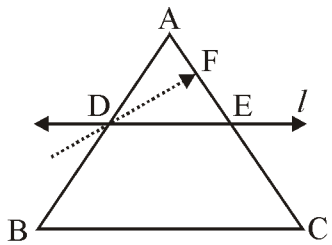
3.2 THEOREM 2

Statement : If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Given : A ΔABC and a line l intersecting AB at D and AC at E, such that $\frac{AD}{DB} = \frac{AE}{EC}$.

To Prove : DE || BC

Proof : If possible let DE be not parallel to BC. Then, there must be another line through D parallel to BC. Let DF || BC. Then, by Basic Proportionality Theorem, we have



$\frac{AD}{DB} = \frac{AF}{FC}$ .....(1)

But $\frac{AD}{DB} = \frac{AE}{EC}$ [Given] .....(2)

From (1) and (2), we have

$\frac{AF}{FC} = \frac{AE}{EC} \Rightarrow \frac{AF}{FC} + 1 = \frac{AE}{EC} + 1$ [Adding 1 on both the sides]

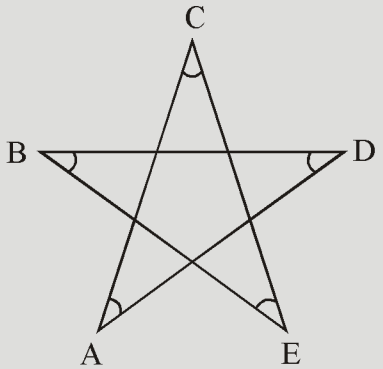
$\Rightarrow \frac{AF + FC}{FC} = \frac{AE + EC}{EC} \Rightarrow \frac{AC}{FC} = \frac{AC}{EC} \Rightarrow FC = EC$

This is only possible, when E and F coincide i.e., DF is the line l itself.

Hence, DE || BC.

Focus Point

(i) The sum of angles of a star (as shown figure) is 180°. i.e. $\angle A + \angle B + \angle C + \angle D + \angle E = 180°$.



(ii) The sum of exterior angles of a polygon is always 360°.

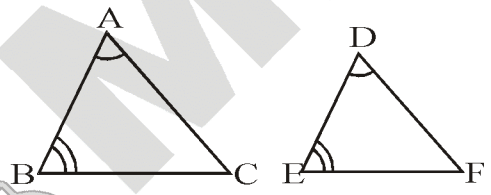
4. CRITERIA FOR SIMILARITY OF TRIANGLES

4.1 AA (ANGLE-ANGLE) AXIOM OF SIMILARITY

If two triangles have two pairs of corresponding angles equal, then the triangles are similar. In the given figure, $\triangle ABC$ and $\triangle DEF$ are such that

$\angle A = \angle D$ and $\angle B = \angle E$.

$\therefore \quad \triangle ABC \sim \triangle DEF$



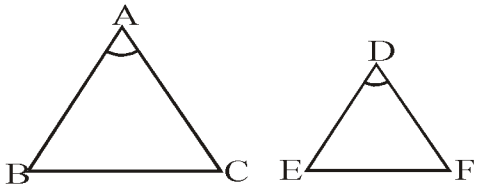
4.2 SAS (SIDE-ANGLE-SIDE) AXIOM OF SIMILARITY

If two triangles have a pair of corresponding angles equal and the sides including them proportional, then the triangles are similar.

In the given fig, $\triangle ABC$ and $\triangle DEF$ are such that

$\angle A = \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$

$\therefore \quad \triangle ABC \sim \triangle DEF.$

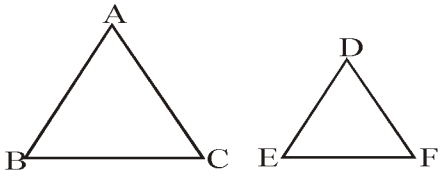


4.3 SSS (SIDE-SIDE-SIDE) AXIOM OF SIMILARITY

If two triangles have three pairs of corresponding sides proportional, then the triangles are similar.

If in $\triangle ABC$ and $\triangle DEF$ we have :

$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$, then $\triangle ABC \sim \triangle DEF$.



5. AREAS OF SIMILAR TRIANGLES

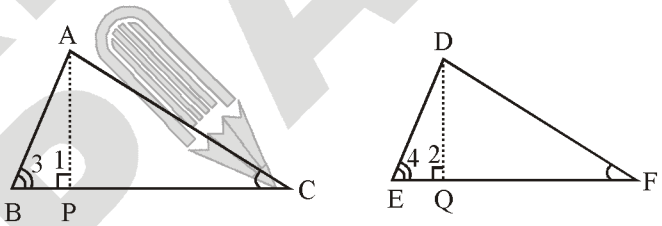
The ratio of the areas of two similar triangles has relation with the ratio of the corresponding sides. The ratio of the areas of two similar triangles is the square of the ratio of their corresponding sides.

Statement : The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Given : $\triangle ABC \sim \triangle DEF$

So, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Also, $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$



To Prove : $\frac{\text{area } \Delta ABC}{\text{area } \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Construction : Through A draw $AP \perp BC$ and through D draw $DQ \perp EF$.

Proof : area $(\triangle ABC) = \frac{1}{2}$ BC × AP and area $(\triangle DEF) = \frac{1}{2}$ EF × DQ.

Thus, $\frac{\text{area } \Delta ABC}{\text{area } \Delta DEF} = \frac{\frac{1}{2} BC \times AP}{\frac{1}{2} EF \times DQ} \Rightarrow \frac{\text{area } \Delta ABC}{\text{area } \Delta DEF} = \frac{BC}{EF} \times \frac{AP}{DQ}$ .....(i)

In $\triangle APB$ and $\triangle DQE$, $\angle 1 = \angle 2 = 90°$ [By construction]

$\angle 3 = \angle 4$ [Given]

$\therefore \triangle APB \sim \triangle DQE$ [AA corollary]

$\therefore \frac{AB}{DE} = \frac{AP}{DQ} = \frac{BP}{EQ} \Rightarrow \frac{AB}{DE} = \frac{AP}{DQ}$ .....(ii)

Also, $\frac{AB}{DE} = \frac{BC}{EF}$ [Given] .....(iii)

From (ii) and (iii), we get $\frac{AP}{DQ} = \frac{BC}{EF}$ .....(iv)

Putting the value of $\frac{AP}{DQ}$ from (iv) in (i), we get

$$\frac{\text{area } \Delta ABC}{\text{area } \Delta DEF} = \frac{BC}{EF} \times \frac{BC}{EF} = \frac{BC^2}{EF^2} \quad .....(v)$$

Similarly, it can also be proved that

$$\frac{\text{area } \Delta ABC}{\text{area } \Delta DEF} = \frac{AB^2}{DE^2} \quad .....(vi)$$

and $$\frac{\text{area } \Delta ABC}{\text{area } \Delta DEF} = \frac{AC^2}{DF^2} \quad .....(vii)$$

From (v), (vi) and (vii), we obtain

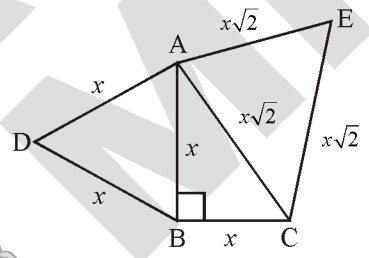
$$\frac{\text{area } \Delta ABC}{\text{area } \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

Example 3

Prove that the area of an equilateral triangle described on a side of a right-angled isosceles triangle is half the area of the equilateral triangle described on its hypotenuse.

Solution :

Given ΔABC in which $\angle ABC = 90°$ and $AB = BC$. ΔABD and ΔACE are equilateral triangles.



To Prove : ar $(\Delta ABD) = \frac{1}{2} \times$ ar (ΔCAE)

Proof : Let $AB = BC = x$ units

$\therefore$ Hypotenuse, $CA = \sqrt{x^2 + x^2} = x\sqrt{2}$ units

Each of the ΔABD and ΔCAE being equilateral, each angle of each one of them is 60°.

$\therefore \Delta ABD \sim \Delta CAE$.

But, the ratio of the area of two similar triangles is equal to the ratio of the squares of their corresponding sides.

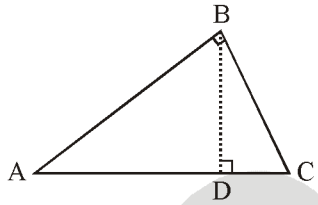
$$\therefore \frac{ar(\Delta ABD)}{ar(\Delta CAE)} = \frac{AB^2}{CA^2} = \frac{x^2}{(x\sqrt{2})^2} = \frac{x^2}{2x^2} = \frac{1}{2}$$

Hence, $ar(\Delta ABD) = \frac{1}{2} \times ar(\Delta CAE)$

6. PYTHAGORAS THEOREM

6.1 THEOREM 1

Pythagoras Theorem : In a right angle triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



Given : Right ΔABC which is right angled at B.

To Prove : $AC^2 = AB^2 + BC^2$

Construction : Draw $BD \perp AC$

Proof : $\Delta ADB \sim \Delta ABC$

$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$ [Sides of similar triangles are proportional]

$\Rightarrow AB^2 = AD \times AC$ .....(1)

Also we have, $\Delta CDB \sim \Delta CBA$

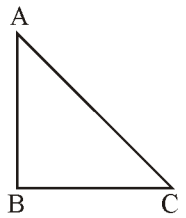
$\Rightarrow \frac{CD}{BC} = \frac{BC}{CA} \Rightarrow BC^2 = CD \times CA$ .....(2)

Adding (1) and (2), we have

$AB^2 + BC^2 = AD \times AC + CD \times AC = AC \times (AD + CD) = AC \times AC = AC^2$.

6.2 THEOREM 2

Converse of Pythagoras Theorem : In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.



Given : ΔABC such that $AB^2 + BC^2 = AC^2$

To Prove : $\angle B = 90°$

Construction : Construct a right triangle PQR, right angled at Q such that PQ = AB and QR = BC

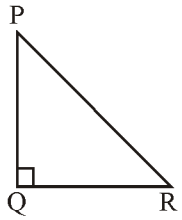
Proof : In right ΔPQR, we have

$PR^2 = PQ^2 + QR^2$ [Pythagoras theorem]

$\Rightarrow PR^2 = AB^2 + BC^2$ .....(1) [$\because$ PQ = AB and QR = BC]

Also, we have $AC^2 = AB^2 + BC^2$ .....(2) [Given]

From (1) and (2), we have



$PR^2 = AC^2 \Rightarrow PR = AC$

In ΔABC and ΔPQR, we have

$AB = PQ$, $BC = QR$ [Construction]

and $AC = PR$ [Proved above]

$\therefore \Delta ABC \cong \Delta PQR$ [SSS congruency criterion]

$\Rightarrow \angle B = \angle Q = 90°$ [Corresponding angles of congruent triangles are equal]

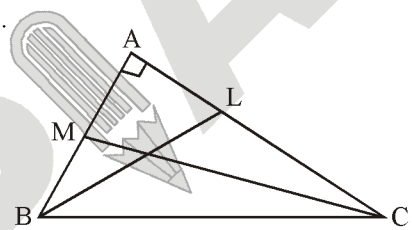
$\Rightarrow \angle B = 90°$ and this is opposite to the first side AC Hence proved.

Example 4

BL and CM are medians of a ΔABC, right angled at A. Prove that $4(BL^2 + CM^2) = 5BC^2$.

Solution :

Given ΔABC in which BL and CM are medians and $\angle A = 90°$.



To Prove : $4(BL^2 + CM^2) = 5BC^2$.

Proof : In ΔBAC, $\angle A = 90°$

$\therefore BC^2 = AB^2 + AC^2$ .....(1)

[By Pythagoras Theorem]

In ΔBAL, $\angle A = 90°$

$\therefore BL^2 = AL^2 + AB^2$ [By Pythagoras Theorem]

$\Rightarrow BL^2 = \left(\frac{1}{2}AC\right)^2 + AB^2$ ($\because$ L is mid point of AC)

$\Rightarrow BL^2 = \frac{1}{4}AC^2 + AB^2$

$\Rightarrow 4BL^2 = AC^2 + 4AB^2$ .....(2)

In ΔCAM, $\angle A = 90°$

$\therefore CM^2 = AM^2 + AC^2$

$\Rightarrow CM^2 = \left(\frac{1}{2}AB\right)^2 + AC^2 \Rightarrow CM^2 = \frac{1}{4}AB^2 + AC^2$

$\Rightarrow 4CM^2 = AB^2 + 4AC^2$ .....(3)

On adding (2) and (3), we get $4(BL^2 + CM^2) = 5(AB^2 + AC^2)$

Hence, $4(BL^2 + CM^2) = 5BC^2$ [Using (1)]

Focus Point

(i) Intersection point of medians is called centroid.

(ii) Medians divide the triangle into 6 equal part.

(iii) Centroid divide the median into 2 : 1.

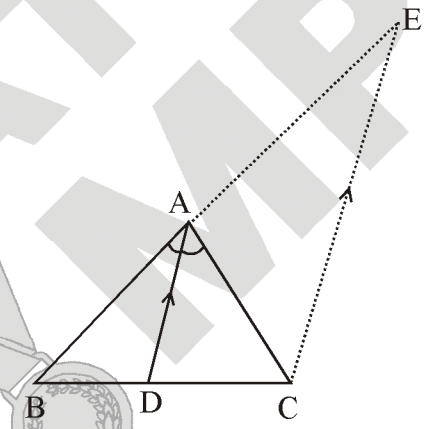
7. ANGLE BISECTOR THEOREM

7.1 INTERIOR ANGLE BISECTOR THEOREM

Given : In ΔABC, AD is the internal bisector of $\angle BAC$ which meets BC at D.

To Prove : $\frac{BD}{DC} = \frac{AB}{AC}$

Construction : Draw CE || DA to meet BA produced at E.



Proof : Since CE || DA and AC is the transversal, we have

$\angle DAC = \angle ACE$ (alternate angles) (1)

and $\angle BAD = \angle AEC$ (corresponding angles) (2)

Since AD is the angle bisector of $\angle A$, $\angle BAD = \angle DAC$ (3)

From (1), (2) and (3), we have $\angle ACE = \angle AEC$

Thus in ΔACE, we have AE = AC

(sides opposite to equal angles are equal)

Now in ΔBCE we have, CE || DA

$\frac{BD}{DC} = \frac{BA}{AE}$ (Thales theorem)

$\Rightarrow \frac{BD}{DC} = \frac{AB}{AC}$ (AE = AC)

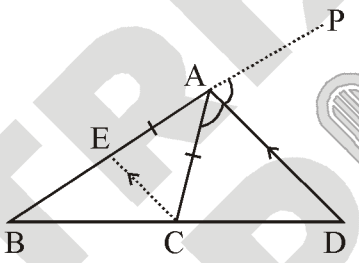
Hence the theorem.

7.2 EXTERIOR ANGLE BISECTOR THEOREM

Given : In ΔABC, AD is the external bisector of ∠BAC and intersects BC produced at D.

To prove : $\frac{BD}{DC} = \frac{AB}{AC}$

Construction : Draw CE || DA meeting AB at E.



Proof : CE || DA and AC is a transversal,

∠ECA = ∠CAD (alternate angles) (1)

Also CE || DA and BP is a transversal

∠CEA = ∠DAP (corresponding angles) (2)

But AD is the bisector of ∠CAP

∠CAD = ∠DAP (3)

From (1), (2) and (3), we have

∠CEA = ∠ECA

(sides opposite to equal angles are equal)

In ΔBDA, we have EC || AD

$\therefore \frac{BD}{DC} = \frac{BA}{AE}$ (Thales theorem)

$\Rightarrow \frac{BD}{DC} = \frac{BA}{AC}$ (AE = AC)

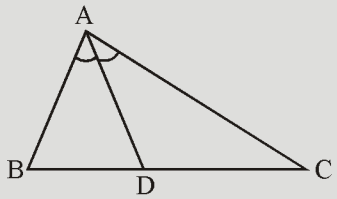
Hence the theorem.

Focus Point

(i) Intersection point of altitudes of a triangle is called orthocentre.

(ii) Intersection point of angle bisectors of a triangle is called incentre.

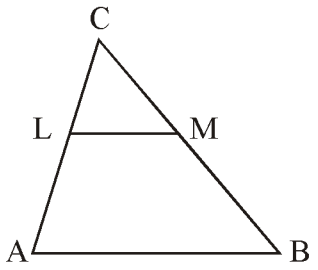
(iii) If AD is angle bisector as shown in figure then $AD^2 = AB \times AC - BD \times CD$.



SOLVED EXAMPLES

SE. 1

LM || AB. If AL = x – 3, AC = 2x, BM = x–2, BC = 2x + 3 find the value of x ?



Ans. In Δ ABC we have LM || AB

$$\frac{AL}{LC}=\frac{BM}{MC}\Rightarrow\frac{AL}{AC-AL}=\frac{BM}{BC-BM}$$

$$\frac{x-3}{2x-(x-3)}=\frac{x-2}{(2x+3)-(x-2)}$$

$$\Rightarrow\frac{x-3}{x+3}=\frac{x-2}{x+5}$$

$\Rightarrow (x-3)(x+5)=(x-2)(x+3)$

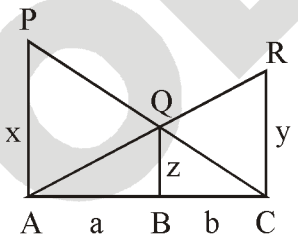
$\Rightarrow x^2+2x-15=x^2+x-6$

$\Rightarrow x=9$

SE. 2

In the given figure PA, QB and RC each is perpendicular to AC such that PA = x, RC = y, QB = z, AB = a and BC = b. Prove that

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{z}.$$



Ans. PA ⊥ AC and QB ⊥ AC ⇒ QB || PA

Thus in Δ PAC, QB || PA

so Δ QBC ~ Δ PAC

$$\therefore\frac{QB}{PA}=\frac{BC}{AC}\Rightarrow\frac{z}{x}=\frac{b}{a+b}$$ ..... (i) (by the property of similar triangle)

In Δ RAC, QB || RC, so Δ QBA ~ Δ RAC

$$\therefore\frac{QB}{RC}=\frac{AB}{AC}\Rightarrow\frac{z}{y}=\frac{a}{a+b}$$ ..... (ii) (by the property of similar triangle)

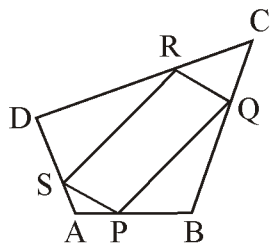
Now from (i) and (ii) we get

$$\frac{z}{x}+\frac{z}{y}=\left(\frac{b}{a+b}+\frac{a}{a+b}\right)=1$$

$$\frac{z}{x}+\frac{z}{y}=1\Rightarrow\left(\frac{1}{x}+\frac{1}{y}=\frac{1}{z}\right)$$

SE. 3

In the adjoining figure. ABCD is a quadrilateral and P, Q, R, S are the points of trisection of the sides AB, BC, CD and DA respectively. Prove that PQRS is a parallelogram.



Ans. Here, ABCD is a quadrilateral. Since R and S are points of trisection of sides CD and DA respectively.

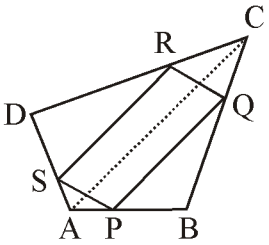
∴ CD = 3CR or CR + DR = 3CR

or DR = 2CR or $\frac{DR}{RC}=\frac{2}{1}$

AD = 3AS or AS + SD = 2AS

or DS = 2AS or $\frac{DS}{SA}=\frac{2}{1}$

$$\Rightarrow\frac{DS}{SA}=\frac{DR}{RC}$$



$\therefore$ By converse of basic proportionality theorem,

In Δ DAC, $\frac{DS}{SA} = \frac{DR}{RC} \Rightarrow SR \parallel AC$

Similarly, PQ || AC

$\therefore$ SR || AC and PQ || AC $\Rightarrow$ SR || PQ

$\Rightarrow$ Similarly one can prove that PS || QR

Hence, PQRS is a parallelogram.

SE. 4

Prove that the line segments joining the mid-points of the adjacent sides of a quadrilateral form a parallelogram.

Ans. Let P, Q, R and S respectively be the mid-points of the sides AB, BC, CD and DA of the quadrilateral ABCD.

Join PQ, QR, RS and SP.

Also, join AC, Since S and R are the mid-points of DA and DC respectively

$\therefore$ DS = SA and DR = RC

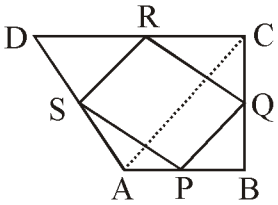
$\Rightarrow \frac{DS}{SA} = 1$ and $\frac{DR}{RC} = 1 \Rightarrow \frac{DS}{SA} = \frac{DR}{RC}$

$\therefore$ In Δ DAC, $\frac{DS}{SA} = \frac{DR}{RC} \Rightarrow SR \parallel AC$

..... (i) (By converse of basic proportionality theorem) Since Q and P are the mid-points of BC and BA respectively.

$\therefore$ BQ = QC and BP = PA

$\Rightarrow \frac{BQ}{QC} = 1$ and $\frac{BP}{PA} = 1 \Rightarrow \frac{BQ}{QC} = \frac{BP}{PA}$



$\therefore$ In Δ BCA, $\frac{BQ}{QC} = \frac{BP}{PA}$

$\Rightarrow$ QP || CA or PQ || AC ..... (ii) (By converse of basic proportionality theorem)

From (i) and (ii), we have PQ || SR

Similarly, PS || QR

Hence, PQRS is a parallelogram.

SE. 5

ABC is a right triangle, right angled at B. If BD is the length of the perpendicular drawn from B to AC. Prove that :

(i) $\Delta ADB \sim \Delta ABC$ and hence $AB^2 = AD \times AC$

(ii) $\Delta BDC \sim \Delta ABC$ and hence $BC^2 = CD \times AC$

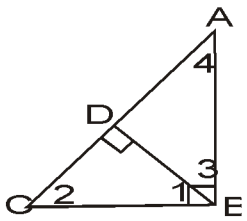
(iii) $\Delta ADB \sim \Delta BDC$ and hence $BD^2 = AD \times DC$

(iv) $\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{BD^2}$

Ans. **Given :** ABC is right angled triangle at B and BD $\perp$ AC

To prove :

(i) $\Delta ADB \sim \Delta ABC$ and hence $AB^2 = AD \times AC$



(ii) $\Delta BDC \sim \Delta ABC$ and hence $BC^2 = CD \times AC$

(iii) $\Delta ADB \sim \Delta BDC$ and hence $BD^2 = AD \times DC$

(iv) $\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{BD^2}$

Proof : (i) In two triangles ADB and ABC, we have :

$\angle BAD = \angle BAC$	(Common)
$\angle ADB = \angle ABC$	(Each is right angle)
$\angle ABD = \angle ACB$	(Third angle)

$\angle ADB \sim \angle ABC$ (AAA Similarity)

Triangle ADB and ABC are similar and so their corresponding sides must be proportional.

$$\frac{AD}{AB}=\frac{DB}{DC}=\frac{AB}{AC} \Rightarrow \frac{AD}{AB}=\frac{AB}{AC}$$

$\Rightarrow AB \times AB = AC \times AD \Rightarrow AB^2 = AD \times AC$

This proves (a).

(ii) Again consider two triangles BDC and ABC, we have

$\angle BCD = \angle ACB$ (Common)

$\angle BDC = \angle ABC$ (Each is right angle)

$\angle DBC = \angle BAC$ (Third angle)

$\therefore$ Triangle are similar and their corresponding sides must be proportional.

i.e., $\Delta BDC \sim \Delta ABC$ $\quad \frac{BD}{AB}=\frac{DC}{BC}=\frac{BC}{AC}$

$$\Rightarrow \frac{DC}{BC}=\frac{BC}{AC} \Rightarrow BC \times BC = DC \times AC$$

$\Rightarrow BC^2 = CD \times AC$. This proves (ii).

(iii) In two triangles ADB and BDC, we have :

$\angle BDA = \angle BDC = 90°$

$\angle 3 = \angle 2 = 90° - \angle 1$

$[\because \angle 1 + \angle 2 = 90°, \angle 1 + \angle 3 = 90°]$

$\angle 1 = \angle 4 = 90° - \angle 2$ $[\because \angle 1 + \angle 2 = 90°,$

$\angle 2 + \angle 4 = 90°]$

$\Delta ADB \sim \Delta BDC$ (AAA criterion of similarity)

$\Rightarrow$ Their corresponding sides must be proportional.

$$\frac{AD}{BD}=\frac{DB}{DC}=\frac{AB}{BC}$$

$$\Rightarrow \frac{AD}{BD}=\frac{BD}{DC} \Rightarrow BD \times BD = AD \times DC$$

$\therefore BD^2 = AD \times DC$

$\Rightarrow$ BD is the mean proportional of AD and DC

(iv) From (i), we have : $AB^2 = AD \times AC$

(ii), we have : $BC^2 = CD \times AC$

(iii) We have : $BD^2 = AD \times DC$

Consider

$$\frac{1}{AB^2}+\frac{1}{BC^2}=\frac{1}{AD \times AC}+\frac{1}{CD \times AC}$$

$$=\frac{1}{AC}\left[\frac{1}{AD}+\frac{1}{DC}\right]$$

$$\frac{1}{AB^2}+\frac{1}{BC^2}=\frac{1}{AC}\left[\frac{DC+AD}{AD \times DC}\right]$$

$$=\frac{1}{AC}\left[\frac{AD+DC}{AD \times DC}\right]=\frac{1}{AC}\left[\frac{AC}{AD \times DC}\right]$$

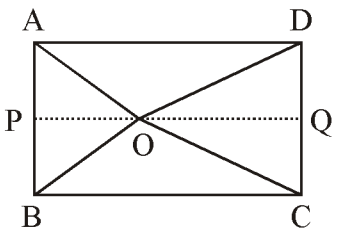
$$=\frac{1}{AD \times DC}=\frac{1}{BD^2} \quad \text{(from (iii))}$$

$$\frac{1}{AB^2}+\frac{1}{BC^2}=\frac{1}{BD^2}$$

Thus we have proved the following

SE. 6

O is any point inside a rectangle ABCD (shown in the figure). Prove that $OB^2 + OD^2 = OA^2 + OC^2$



Ans. Through O, draw PQ||BC so that P lies on AB and Q lies on DC.

Now, PQ||BC

Therefore, $PQ \perp AB$ and $PQ \perp DC$

$[\angle B = 90°$ and $\angle C = 90°]$

So, $\angle BPQ = 90°$ and $\angle CQP = 90°$

Therefore, BPQC and APQD are both rectangles.

Now, from ΔOPB,

$OB^2 = BP^2 + OP^2$ ....(i)

Similarly, from ΔODQ,

$OD^2 = OQ^2 + DQ^2$ ....(ii)

From ΔOQC, we have

$OC^2 = OQ^2 + CQ^2$ ...(iii)

And from ΔOAP, we have

$OA^2 = AP^2 + OP^2$ ....(iv)

Adding (i) and (ii)

$OB^2 + OD^2 = BP^2 + OP^2 + OQ^2 + DQ^2$

$= CQ^2 + OP^2 + OQ^2 + AP^2$

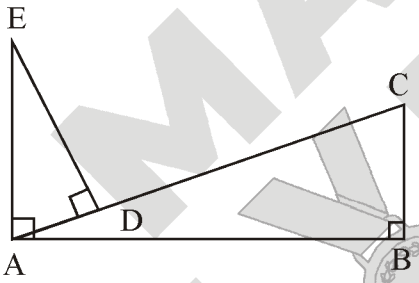
[As BP = CQ and DQ = AP]

$= CQ^2 + OQ^2 + OP^2 + AP^2$

$= OC^2 + OA^2$ [From (iii) and (iv)]

SE. 7

In the given figure, $BC \perp AB$, $AE \perp AB$ and $DE \perp AC$. Prove that DE.BC = AD.AB.



Ans. In ΔABC and ΔEDA, We have

$\angle ABC = \Delta ADE$ [Each equal to 90°]

$\angle ACB = \angle EAD$ [Alternate angles]

$\therefore$ By AA Similarity

$\Delta ABC \sim \Delta EDA$

$$\Rightarrow \frac{BC}{AB} = \frac{AD}{DE}$$

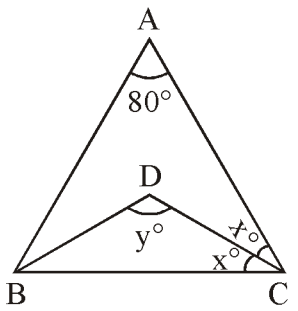
$\Rightarrow$ DE.BC = AD.AB.

Space for Notes :

EXERCISE – I

ONLY ONE CORRECT TYPE

1. In the given figure $\angle A = 80°$, $\angle B = 60°$, $\angle C = 2x°$ and $\angle BDC = y°$, BD and CD bisect angles B and C respectively. The values of x and y, respectively, are

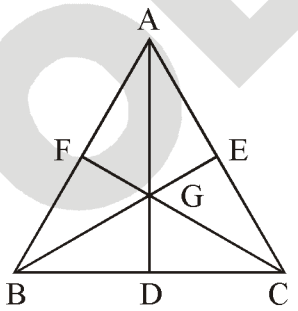


(A) 15°, 70° (B) 10°, 160°
(C) 20°, 130° (D) 20°, 125°

2. If a + b + c = 2s, then the value of $(s - a)^2 + (s - b)^2 + (s - c)^2$ will be :
(A) $s^2 + a^2 + b^2 + c^2$ (B) $a^2 + b^2 + c^2 - s^2$
(C) $s^2 - a^2 - b^2 - c^2$ (D) $4s^2 - a^2 - b^2 - c^2$

3. If D is a point on the side BC = 12 cm of a ΔABC such that BD = 9 cm and $\angle ADC = \angle BAC$, then the length of AC is equal to :
(A) 9 cm (B) 6 cm
(C) $6\sqrt{3}$ cm (D) 3 cm

4. In ΔABC medians BE and CF intersect at G. If the straight line AGD meets BC at D in such a way that GD = 1.5 cm, then the length of AD is :



(A) 2.5 cm (B) 3.0 cm
(C) 4.00 cm (D) 4.5 cm

5. The side of an equilateral triangle is $20\sqrt{3}$ cm. The numerical value of the radius of the circle circumscribing the triangle is :
(A) 20 cm (B) $20\sqrt{3}$ cm
(C) 20 π cm (D) $\frac{20}{\pi}$

6. If ΔABC is a right angled triangle with $\angle A = 90°$, AN is perpendicular to BC, BC = 12 cm and AC = 6 cm, then the ratio of $\frac{\text{area } \Delta ANC}{\text{area } \Delta ABC}$:
(A) 1 : 3 (B) 1 : 2
(C) 1 : 4 (D) 1 : 8

7. The area of the largest triangle inscribed in a semi - circle of radius R is :
(A) $2R^2$ (B) R^2
(C) $\frac{1}{2}R^2$ (D) $\frac{3}{2}R^2$

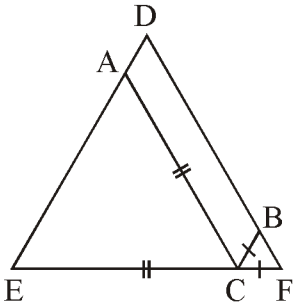
8. In a triangle ABC, then sum of the exterior angles at B and C is equal to :
(A) $180° - \angle BAC$ (B) $180° + \angle BAC$
(C) $180° - 2\angle BAC$ (D) $180° + 2\angle BAC$

9. In ΔABC, $\angle B = 3x$, $\angle A = x$, $\angle C = y$ and $3y - 5x = 30$, then the triangle is ;
(A) isosceles (B) equlateral
(C) right angled (D) scalene

10. The internal bisectors of $\angle B$ and $\angle C$ of ΔABC meet at O. If $\angle A = 80°$, then $\angle BOC$ is :
(A) 50° (B) 100°
(C) 130° (D) 160°

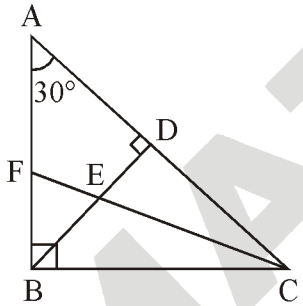
11. The areas of two similar triangles are 12 cm^2 and 48 cm^2. If the height of the smaller one is 2.1 cm, then the corresponding height of the bigger triangle is :
(A) 12.6 cm (B) 8.4 cm
(C) 4.2 cm (D) 1.05 cm

12. In a ΔDEF shown in given figure, points A, B and C are taken on DE, DF and EF respectively, such that EC = AC and CF = BC. If ∠D = 40°, then what is ∠ACB in degrees ?



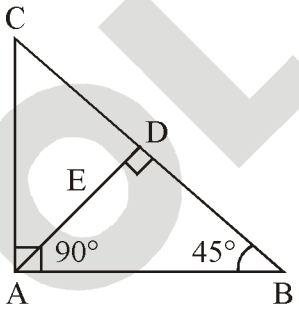
(A) 140 (B) 70
(C) 100 (D) None of these

13. AB ⊥ BC, BD ⊥ AC and CE bisects ∠C. If A = 30°. Then, what is ∠CED ?



(A) 30° (B) 60°
(C) 45° (D) 65°

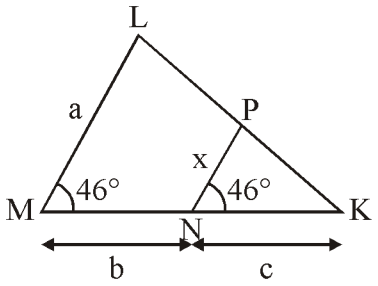
14. In ΔABC, ∠A = 90°, AD ⊥ BC and ∠B = 45°. If AB = x, then the value of AD in terms of x is :



(A) $\frac{x}{2}$ (B) $\frac{\sqrt{x}}{2}$

(C) $\frac{x}{\sqrt{2}}$ (D) $\sqrt{\frac{x}{2}}$

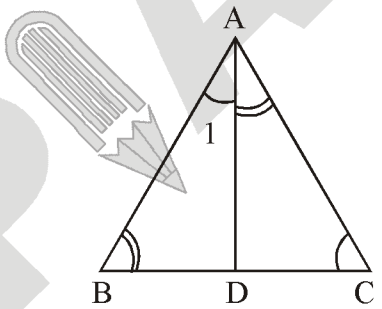
15. Express x in terms of a, b, and c.



(A) $x = \frac{ac}{b+c}$ (B) $x = \frac{bc}{a+c}$

(C) $x = \frac{b+c}{ac}$ (D) $x = \frac{ab}{a+c}$

16. In ΔABC, if AD ⊥ BC and $AD^2 = BD \times DC$. Then find the angle ∠BAC = ?



(A) 60°
(B) 90°
(C) 30°
(D) None of this

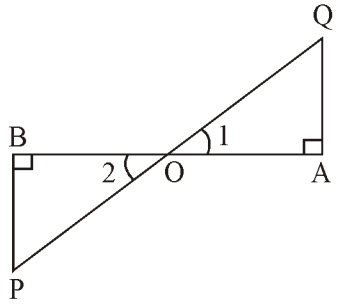
17. In a trapezium ABCD, AB || CD and DC = 3AB. EF || AB intersects DA and CB at E and F such that $\frac{BF}{FC} = \frac{2}{3}$. Then 3DC =

(A) 4EF (B) 2EF
(C) 5EF (D) EF

18. The corresponding altitude of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of their areas.

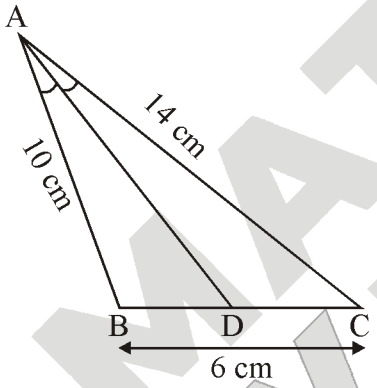
(A) 2 : 1 (B) 3 : 2
(C) 4 : 9 (D) 8 : 16

19. In the given figure PB and QA perpendicular to segment AB. If PO = 5cm, QO = 7cm and area $\Delta POB = 150cm^2$, find the area of ΔQOA.



(A) 294 cm^2 (B) 200 cm^2
(C) 269 cm^2 (D) 225 cm^2

20. In the given figure AD is the bisector of $\angle BAC$. If AB = 10 cm, AC = 14 cm and BC = 6 cm. Then find BD and DC ?



(A) $\frac{2}{7}, \frac{5}{7}$ (B) $\frac{5}{2}, \frac{7}{2}$
(C) $\frac{14}{5}, \frac{10}{7}$ (D) $\frac{14}{5}, \frac{5}{7}$

21. If corresponding sides of two similar triangles are in the ratio 4 : 5, then corresponding medians of the triangles are in the ratio 4 : k. Find k.

(A) 5 (B) 6
(C) 12 (D) 10

22. P and Q are the points on sides AB and AC respectively of a ΔABC, such that PQ || BC. If AP : PB = 2 : 3 and AQ = 4 cm, then the length of AC is.....cm.

(A) 5 (B) 8
(C) 11 (D) 10

23. XY is drawn parallel to the base BC of ΔABC cutting AB at X and AC at Y. If AB = 4BX and YC = 2 cm, then AY is ..... cm.

(A) 6 (B) 7
(C) 15 (D) 13

24. In ΔABC, if DE || BC, AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, then the value of x is.

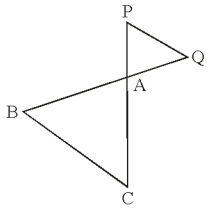
(A) 2 (B) 0
(C) 1 (D) 4

25. The altitude of an equilateral triangle having the length of its side 10 cm is $k\sqrt{3}$ cm. Find k.

(A) 10 (B) 8
(C) 15 (D) 5

PARAGRAPH TYPE

Comprehension : $\Delta ACB \sim \Delta APQ$. If BC = 10 cm, PQ = 5 cm, BA = 6.5 cm and AP = 2.8 cm.



26. Find the length of CA ?

(A) 5.6 cm (B) 6 cm
(C) 6.5 cm (D) 5 cm

27. Find the area of ΔACB is :

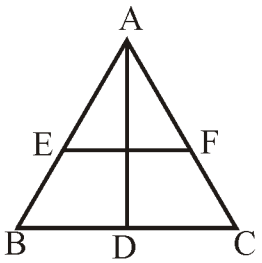
(A) 16.96 (B) 17
(C) 16 (D) None of these

28. Find the ratio of the area of triangle ACB and APQ.

(A) 1 : 4 (B) 4 : 1

(C) 2 : 1 (D) 3 : 1

In figure :-



$\angle ADB = 90^\circ$ and $AB = AC$, $BC = 10cm$, $AC = 12cm$.

29. Find EF

(A) 5 (B) 10

(C) 15 (D) 13

30. Find area of ΔABC.

(A) $4\sqrt{56}$ (B) $4\sqrt{14}$

(C) $2\sqrt{56}$ (D) $2\sqrt{14}$

31. Find AD

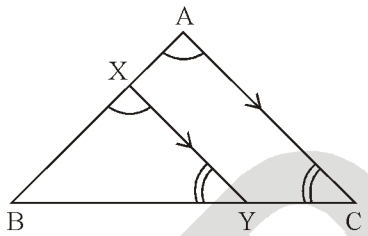
(A) $\sqrt{119}$ (B) 119

(C) 10 (D) $2\sqrt{11}$

MATCH THE COLUMN TYPE

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. Only One entries of column-I may have the matching with the some entries of column-II and one entry of column-II Only one matching with entries of column-I

32. In the figure, the line segment XY is parallel to the sideAC of ΔABC and it divides the triangle into two parts of equal areas, then match the column



Column – I	**Column – II**
(A) AB : XB	(P) $\sqrt{2}$: 1
(B) ar (ΔABC) : ar (ΔXBY)	(Q) 2 : 1
(C) AX : AB	(R) $\left(\sqrt{2}-1\right):\sqrt{2}$
(D) ∠X : ∠A	(S) 1 : 1

(A) A–P, B–Q, C–R, D–S

(B) A–Q, B–P, C–R, D–S

(C) A–P, B–Q, C–S, D–R

(D) A–P, B–R, C–Q, D–S

33. Match the Column :

Column - I	**Column - II**
(A) In ΔABC and ΔPQR $\frac{AB}{PQ}=\frac{AC}{PR}, \angle B = \angle P$	(P) AA similarity
(B) In ΔABC and ΔPQR, $\angle A = \angle P, \angle B = \angle Q$	(Q) SAS similarity
(C) In ΔABC and ΔPQR $\frac{AB}{PQ}=\frac{AC}{PR}=\frac{BC}{QR}$	(R) SSS similarity

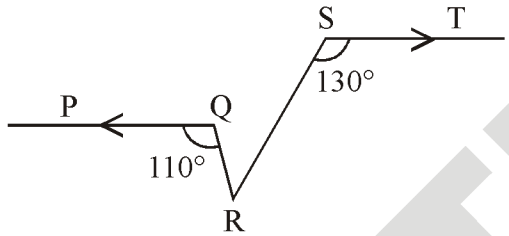
(D) In $\Delta ACB, DE || BC$ (S) BPT

$\Rightarrow \frac{AD}{BD} = \frac{AE}{CE}$

(A) A–P, B–Q, C–R, D–S
(B) A–Q, B–P, C–R, D–S
(C) A–P, B–Q, C–S, D–R
(D) A–Q, B–R, C–P, D–S

ANALYTICAL TYPE

34. In the given figure PQ || ST, ∠PQR = 110°, ∠RST = 130° then value of ∠QRS is

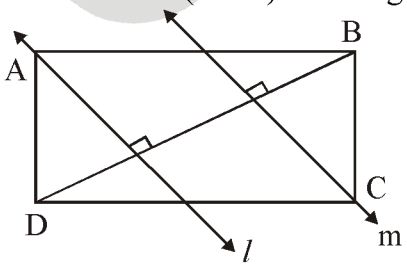


(A) 20° (B) 50°
(C) 60° (D) 70°

35. E and F are respectively, the mid points of the sides AB and AC of ΔABC and the area of the quadrilateral BEFC is k times the area of ΔABC. The value of k is :

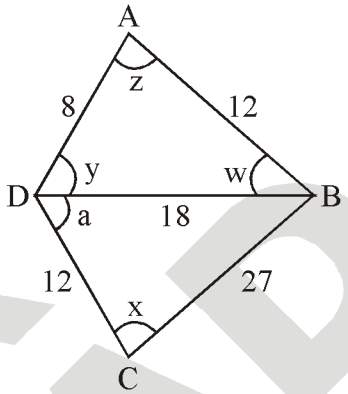
(A) $\frac{1}{2}$ (B) 3
(C) $\frac{3}{4}$ (D) 4

36. In the figure, DB is diagonal of rectangle ABCD and line *l* through A and line m through C divide DB in three equal parts each of length 1 cm and are perpendicular to DB. Area (in cm^2) of rectangle ABCD is :



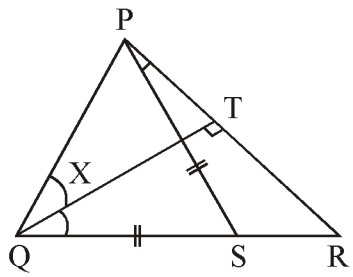
(A) $2\sqrt{2}$ (B) $2\sqrt{3}$
(C) $3\sqrt{2}$ (D) $3\sqrt{3}$

37. In the quadrilateral ABCD :



(A) x = y, a = z (B) x = z, a = y
(C) x = z, a = – y (D) x = y, a = w

38. In the following figure QT ⊥ PR and QS = PS. If ∠TQR = 40° and ∠RPS = 20° the value of *x* is



(A) 80° (B) 25°
(C) 15° (D) 35°

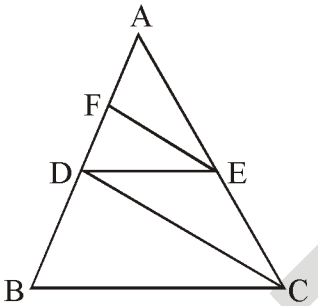
EXERCISE – II

VERY SHORT ANSWER TYPE

1. ABCD is a trapezium with AB || DC. If ΔAED is similar to ΔBEC. Prove that AD = BC.

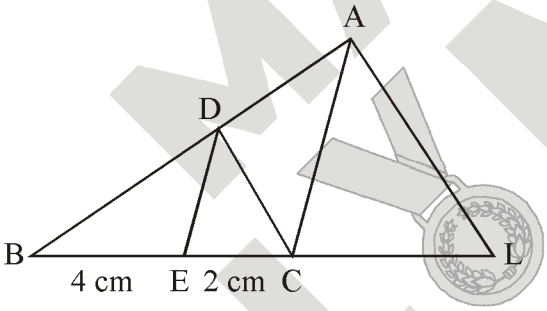
2. ABC is a triangle right angled at C. If p^2 is the length of perpendicular from C to AB and $AB = c^2$, $BC = a^2$ and $CA = b^2$, show that $(pc)^2 = (ab)^2$.

3. In figure, DE || BC and CD || EF. Prove that $AD^2 = AB \times AF$.

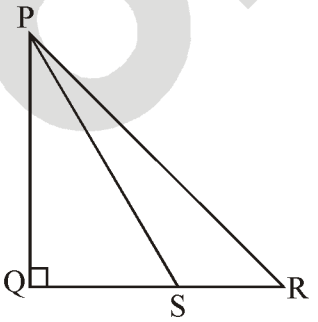


4. ABC is a right triangle right angled at C and AC = $\sqrt{3}$ BC. Prove that $\angle ABC = 60°$

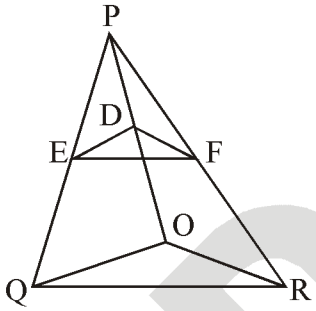
5. In figure, CD || LA and DE || AC. Find CL.



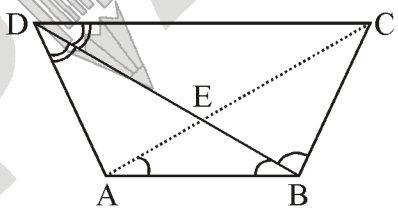
6. PQR is right angled triangle, having $\angle Q = 90°$. If QS = SR, show that $PR^2 = 4PS^2 - 3PQ^2$.



7. In the given figure, DE || OQ and DF || OR. Show that EF || QR.



8. If the diagonal BD of a quadrilateral ABCD bisects both $\angle B$ and $\angle D$, prove that $\frac{AB}{BC} = \frac{AD}{CD}$.

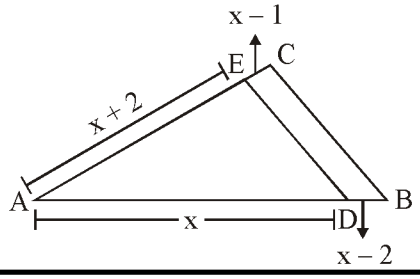


9. Prove that the area of the equilateral described on the side of a square is half the area of the equilateral triangle described on its diagonal.

10. If ΔABC is such that AB = 3 cm, BC = 2 cm and CA = 2.5 cm. If $\Delta DEF \sim \Delta ABC$ and EF = 4 cm, then perimeter of ΔDEF is.

SHORT ANSWER TYPE

1. In the given figure, DE || BC. If AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, find the value of x.



2. ABCD is a parallelogram, P is a point on side BC and DP, when produced meets AB produced at L. Prove that

(i) $\frac{DP}{PL} = \frac{DC}{BL}$ (ii) $\frac{DL}{DP} = \frac{AL}{DC}$

3. In a ΔABC, AD is the bisector of ∠A, meeting side BC at D. If AC = 4.2cm, DC = 6cm, BC = 10cm, find AB.

4. In ΔABC, if AD is the bisector of ∠A, Prove that $\frac{\text{Area}(\Delta ABD)}{\text{Area}(\Delta ACD)} = \frac{AB}{AC}$.

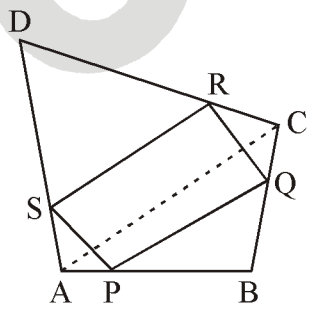
5. In ΔABC, D is the mid point of BC and ED is the bisector of ∠ADB and EF is drawn parallel to BC cutting AC in F. Prove that ∠EDF is a right angle.

LONG ANSWER TYPE

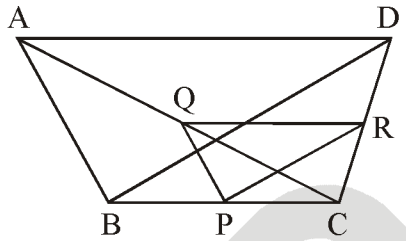
1. Let X be any point on the side BC of a ΔABC. If XM, XN are drawn parallel to BA and CA meeting. CA, BA in M, N respectively, MN meets BC produced in T, prove that TX^2 = (TB) (TC).

2. Let ABC be a triangle D and E be two points on side AB such that AD = BE. If DP || BC and EQ || AC, then prove that PQ || AB.

3. ABCD is a quadrilateral. P, Q, R and S are the points of trisection of sides AB, BC, CD, and DA respectively and are adjacent to A and C. Prove that PQRS is a parallelogram.



4. ΔABC and ΔDBC line on the same side of the base BC. From a Point P on BC, PQ || AB and PR || BD are drawn. They meet AC in Q and DC in R respectively. Prove that QR || AD.



5. Two poles of height 'a' metres and 'b' metres are 'p' metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ meters.

TRUE / FALSE TYPE

1. In obtuse angle triangle the orthocentre will lie out side the triangle.
2. Medians divide the triangle into 6 equal parts.
3. Intersection point of medians is called centroid.
4. If in ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar if $\angle B = \angle D$.
5. Two figure having the same shape but not necessarily the same size are called similar figure.

FILL IN THE BLNAKS TYPE

1. Centroid divides the median into _____ .
2. All circle are _____ .
3. Two polygons of the same number of sides are similar, if their corresponding angles are _____ and their corresponging sides are _____ .
4. For two triangles, if corresponding angles are equal, then the two triangles are similar, this is called _____ similarity.
5. All _____ triangles are similar.

NUMERICAL PROBLEMS

1. ΔABC and ΔCDE are two equilateral triangles such that D is the mid-point of BC. The ratio of the areas of ΔCDE and ΔABC is 1 : k then k =
2. If ΔABC is an equilateral triangle such that $AD \perp BC$, $AD^2 = kDC^2$, then k is :
3. If each side of a rhombus is 10 cm and one of its diagonals is 16 cm, then the length of the other diagonal is k cm. Find k.
4. If E is a point in side CA of an equilateral ΔABC such that $BE \perp AC$, then $AB^2 + BC^2 + CA^2 = kBE^2$. Find k.
5. If in an isosceles triangle a is the length of the base and b the length of one of the equal side , then its area is $\frac{a}{k}\sqrt{kb^2 - a^2}$. Find k.

Space for Notes :

Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	B	B	D	A	C	B	B	C	C	C	C	B	C	A
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
B	C	C	A	B	A	D	A	D	D	A	A	B	A	B
31	32	33	34	35	36	37	38							
A	A	B	C	A	C	A	C							

EXERCISE II

VERY SHORT ANSWER TYPE

5. 3 **10.** 15

SHORT ANSWER TYPE

1. 4 **3.** 2.8 cm

TRUE FALSE TYPE

1. T **2.** T **3.** T **4.** T **5.** T

FILL IN THE BLANKS

1. 2 : 1 **2.** similar **3.** equal, proportioanl **4.** AAA **5.** equilateral

NUMERICAL TYPE

1. 4 2. 3 3. 12 4. 4 5. 4

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : TRIANGLES)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

COORDINATE GEOMETRY

7

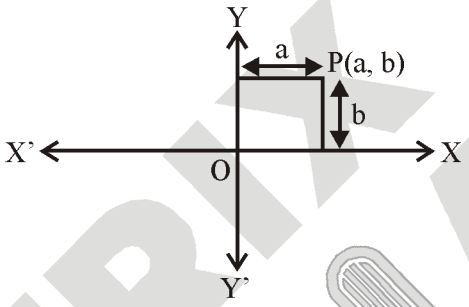
Concepts

INTRODUCTION

Co–ordinate geometry is that part of geometry, in which the points on the plane are represented by ordered pairs of real numbers. Straight lines and curves are represented by algebraic equations.

Thus coordinate geometry or analytical geometry is that branch of mathematics in which geometrical problems are solved with the help of algebra.

1. COORDINATE AXIS AND CARTESIAN SYSTEM



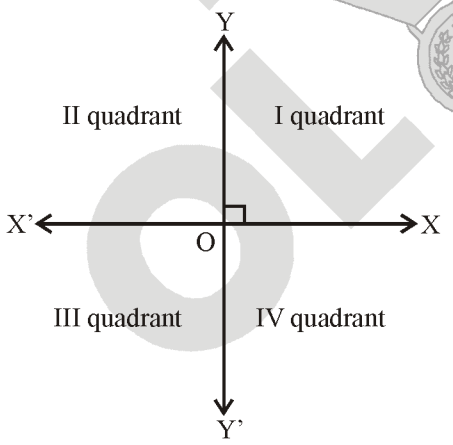
Two perpendicular lines XX' and YY' on a plane intersecting at a point O are know as rectangular axis and plane is called Cartesian plane. Point O is known as origin.

XX' is called x–axis.

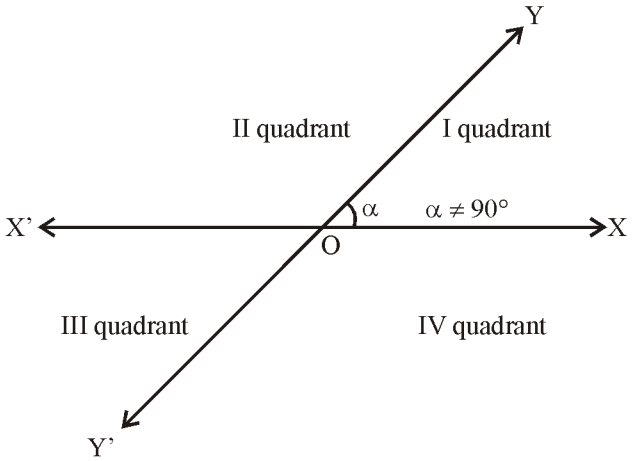
YY' is called y–axis.

The position of a point in a plane is determined with the reference to two intersecting straight lines called the coordinate axis and their point of intersection is called the origin of co–ordinates.

If these two axis of reference out each other at right angle, they are called rectangular axis otherwise they are called oblique axis. The axis divide the coordinate plane in four quadrants.



(rectangular axis)



(oblique axis)

2. RECTANGULAR CARTESIAN COORDINATES OF A POINT

Let XOX' and YOY' be two mutually perpendicular axis in the plane of paper intersecting at O. Let P be any point in the plane. Draw PM perpendicular to OX, then lengths OM and PM are called the rectangular Cartesian coordinates of P.

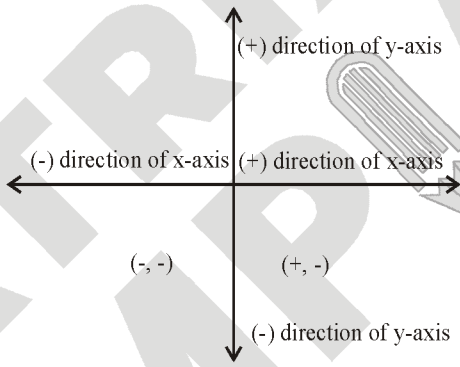
Notes: (i) The ordinate of every point on x–axis is 0.

(ii) The abscissa of every point on y–axis is 0.

(iii) Coordinates of origin are O (0, 0).

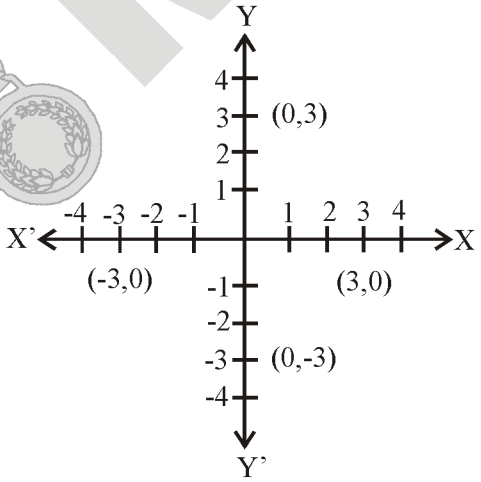
If a point P is situated at a distance 'b' from x–axis and at a distance 'a' from y–axis, then its co–ordinates are represented by ordered pair (a, b). Every ordered pair represents a unique point on the plane. Distance of point P from x–axis is ordinate and that from y–axis is abscissa. Also, we have to keep in mind (a, b) ≠ (b, a).

In the Cartesian co–ordinates, points lying on right side of origin have x co–ordinate positive, while points lying on left side of origin have x co–ordinates negative. In the same way, points lying above origin have y co–ordinate positive and below origin have y co–ordinate negative.



Example 1

Find the co–ordinate of a point at a distance of 3 units from origin which lie on (i) x–axis (ii) y–axis.



Solution :

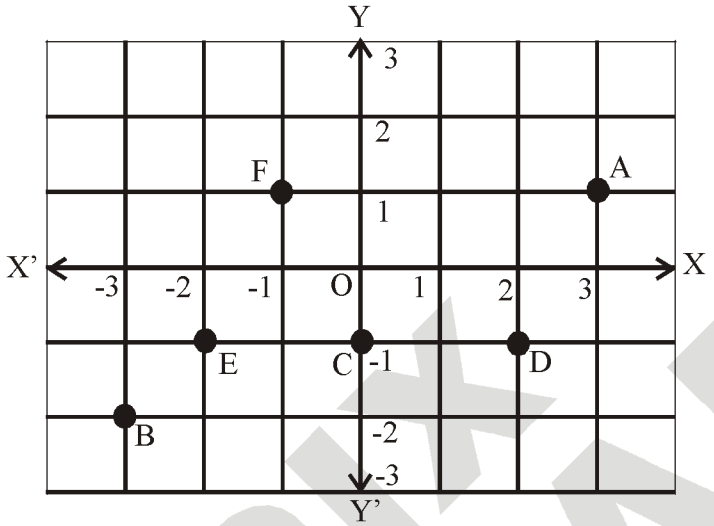
(i) 3 units distance of a point from origin can be in positive direction of x–axis in negative direction of x–axis.

∴ co–ordinates of a point at a distance of 3 units on x–axis from origin is either (–3, 0) or (3, 0).

(ii) In the same manner co–ordinate of a point on y–axis is (0, 3) or (0, –3).

Example 2

Find the co–ordinates of the points as shown in the figure :



Solution :

Co–ordinate of A = (3, 1)

Co–ordinate of B = (–3, –2)

Co–ordinate of C = (0, –1)

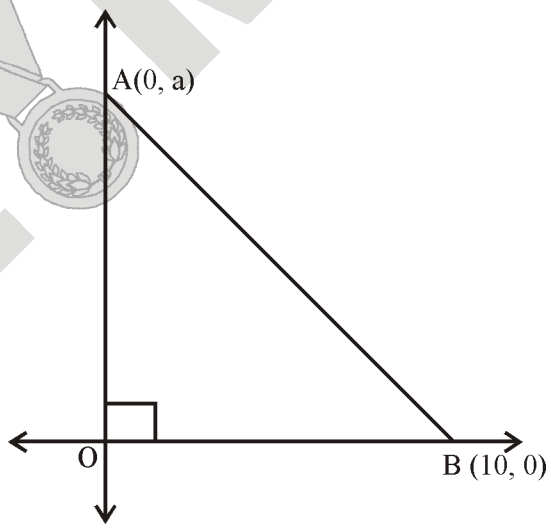
Co–ordinate of D = (2, –1)

Co–ordinate of E = (–2, –1)

Co–ordinate of F = (–1, 1)

Example 3

In the figure, if area of the triangle AOB is 100 square units, find the co–ordinates of A.



Solution :

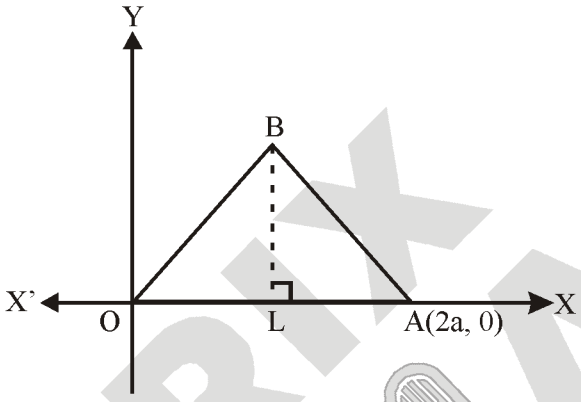
Area of triangle AOB = $\frac{1}{2} \times OB \times OA$

$\frac{1}{2} \times 10 \times a = 100 \Rightarrow a = 20$

$\therefore$ Co–ordinate of A is (0, 20).

Example 4

In the given figure, find the co–ordinates of the vertices of an equilateral triangle of side 2a.



Solution :

Since OAB is an equilateral triangle of side 2a. Therefore OA = AB = OB = 2a

Let BL be the perpendicular from B on OA. Then

OL = LA = a

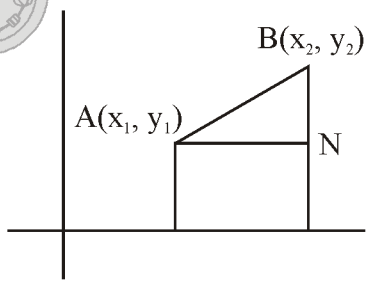
In ΔOLB, $OB^2 = OL^2 + LB^2$

$\Rightarrow (2a)^2 = a^2 + LB^2$

$\Rightarrow LB = \sqrt{3}\,a$

$\therefore$ coordinates of O are (0, 0) A(2a, 0) and B $(a, \sqrt{3}\,a)$

3. DISTANCE BETWEEN TWO POINTS

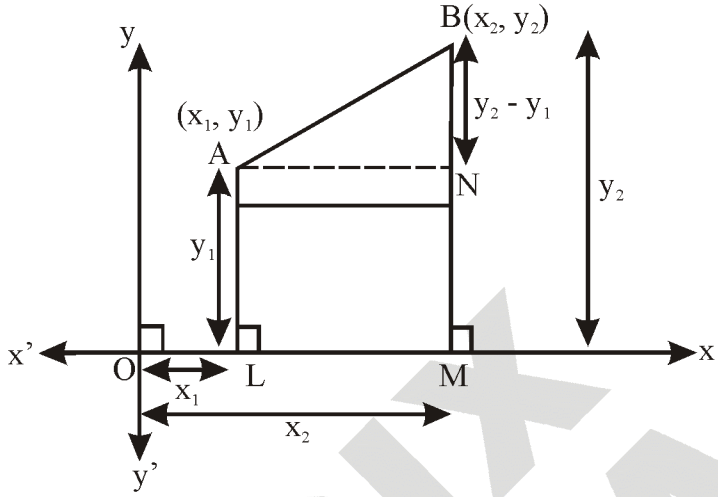


Let two points be $A(x_1, y_1)$ and $B(x_2, y_2)$, then distance between them is given by $|AB| = \sqrt{\{(x_2 - x_1)^2 + (y_2 - y_1)^2\}}$

or, Distance between two points

$= \sqrt{(\text{Difference of x coordinates})^2 + (\text{Difference of y coordinates})^2}$

3.1 PROOF OF DISTANCE FORMULA



Let A (x_1, y_1) and B (x_2, y_2) be any two points in the plane (assume points in I quadrant)

Draw AL ⊥ OX and BM ⊥ OX.

Also, draw AN ⊥ BM

Then $OL = x_1$, $OM = x_2$, $AL = y_1$, $BM = y_2$

$AN = LM = OM - OL = x_2 - x_1$

$BN = BM - NM = y_2 - y_1$

Now, in right ΔANB

$AB^2 = AN^2 + NB^2$

$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)$

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Focus Point

(i) The distance of point P(x, y) from origin O(0, 0) is given by : $|OP| = \sqrt{x^2 + y^2}$

(ii) If AB is parallel to x–axis, then $y_1 = y_2$ and so $|AB| = \sqrt{(x_2 - x_1)^2} = |x_2 - x_1|$

(iii) If AB is parallel to y–axis, then $x_1 = x_2$ and so $|AB| = \sqrt{(y_2 - y_1)^2} = |y_2 - y_1|$

Example 5

Distance of any point P(3, 2) from any point Q is $\sqrt{2}$ and ordinate of Q is half that of its abscissa. Then find its co–ordinates.

Solution :

Let ordinate of Q be y.

$\therefore$ Abscissa of Q is 2y.

Co–ordinates of Q are (2y, y).

Now, distance between (3, 2) and (2y, y) is

$$\sqrt{(3-2y)^2+(2-y)^2}=\sqrt{2}$$

$\Rightarrow (3-2y)^2+(2-y)^2=2$

$\Rightarrow 5y^2-16y+11=0$

On solving, we get $y=\frac{11}{5}$ or $y=1$

$\therefore$ Co–ordinates of Q are $\left(\frac{22}{5},\frac{11}{5}\right)$ or (2, 1)

Example 6

If point P(x, y) is situated on a circle whose centre is (3, –2) and whose radius is 3, then prove that $x^2+y^2-6x+4y+4=0$.

Solution :

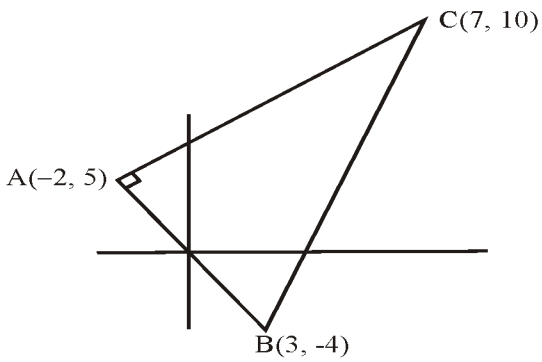
Distance of centre from P(x, y) is 3 and centre O(3, –2)

$\therefore \sqrt{(x-3)^2+(y+2)^2}=3$

On squaring both the sides and solving, we get $x^2+y^2-6x+4y+4=0$

Example 7

Prove that the points (–2, 5), (3, –4), (7, 10) are the vertices of an isosceles triangle. What more information you can give about the triangle ?



Solution :

Let three given points A, B, C.

$AB^2 = (-2 - 3)^2 + (5 + 4)^2 = 106$

$BC^2 = (3 - 7)^2 + (-4 - 10)^2 = 212$

$AC^2 = (-2 - 7)^2 + (5 - 10)^2 = 106$

$\therefore$ AB = $\sqrt{106}$ = AC and

$BC^2 = 212 = 106 + 106 = AB^2 + AC^2$

$\therefore$ ABC is a right angled isosceles triangle.

Example 8

Find the distance between the points $A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$

Solution :

$$AB = \sqrt{\left(at_2^2 - at_1^2\right)^2 + \left(2at_2 - 2at_1\right)^2}$$

$$AB = \sqrt{a^2\left(t_2 - t_1\right)^2\left(t_2 + t_1\right)^2 + 4a^2\left(t_2 - t_1\right)^2}$$

$$AB = a\,(t_2 - t_1)\,\sqrt{\left(t_2 + t_1\right)^2 + 4}$$

Example 9

Find a point a on x–axis which is equidistant from A(2, –5) and B(–2, 9).

Solution :

Let the point on x–axis be P(x, 0)

$\therefore$ PA = PB

$$\Rightarrow \sqrt{(x-2)^2 + (0+5)} = \sqrt{(x+2)^2 + (0-9)^2}$$

$\Rightarrow (x - 2)^2 + 25 = (x + 2)^2 + 81$

$\Rightarrow x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$

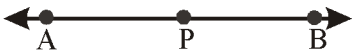
$\Rightarrow -8x = 56 \Rightarrow x = -7$

$\therefore$ point is (–7, 0).

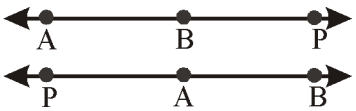
4. SECTION FORMULAS

If P be any point on the line AB between A and B, then we say that P divides line segment AB internally in the ratio AP : PB.

Also, if P be any point on the line but not between A and B (P may be to the right or the left of the points A, B) then P divides AB externally in the ratio AP : PB.



(Internal division)

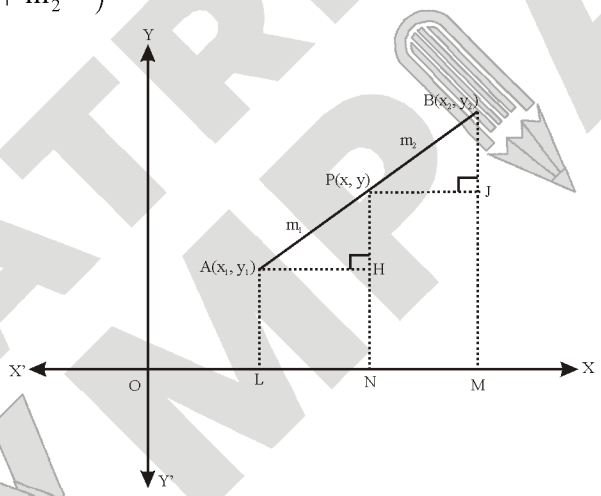


(External division)

4.1 INTERNAL DIVISION

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points and point P(x, y) divides the line segment AB in the ratio $m_1 : m_2$ internally, then

$$(x, y) \equiv \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$



Proof : The given points are $A(x_1, y_1)$ and $B(x_2, y_2)$. Let us assume that points A and B are both in I quadrant. Since P(x, y) divides divides AB internally in ration $m_1 : m_2$ i.e. $AP : PB = m_1 : m_2$.

Now, Draw AL, BM and PN perpendiculars to x–axis.

Draw, $AH \perp PN$ and $PJ \perp BM$, then

$OL = x_1$, $ON = x$, $OM = x_2$

$AL = y_1$, $PN = y$, $BM = y_2$

$\therefore\ AH = LN = ON - OL = x - x_1$

$PJ = NM = OM - ON = x_2 - x$

$PH = PN - HN = PN - AL = y - y_1$

$BJ = BM - JM = BM - PN = y_2 - y$

clearly, ΔAHP and ΔPJB are similar.

$$\therefore\ \frac{AH}{PJ} = \frac{PH}{BJ} = \frac{AP}{PB}$$

or $\dfrac{x-x_1}{x_2-x}=\dfrac{y-y_1}{y_2-y}=\dfrac{m_1}{m_2}$

Now, $\dfrac{x-x_1}{x_2-x}=\dfrac{m_1}{m_2} \Rightarrow m_2x - m_2x_1 = m_1x_2 - m_1x$

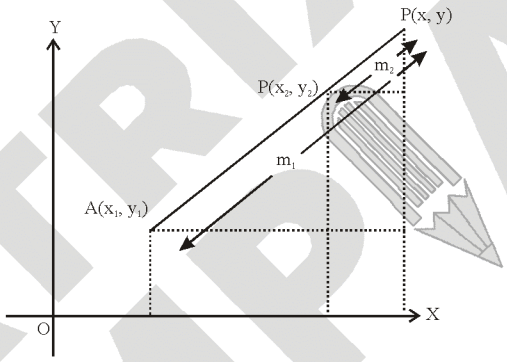
$\Rightarrow (m_1 + m_2)x = m_2x_1 + m_1x_2 \qquad \Rightarrow x = \dfrac{m_2x_1+m_1x_2}{m_1+m_2}$

and $\dfrac{y-y_1}{y_2-y}=\dfrac{m_1}{m_2} \Rightarrow (m_1 + m_2)\, y = m_2y_1 + m_1\, y_2 \qquad \Rightarrow y = \dfrac{m_2y_1+m_1y_2}{m_1+m_2}$

4.2 EXTERNAL DIVISION

Let P (x, y) divide the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio $m_1 : m_2$, then

$$(x, y) \equiv \left(\frac{m_1x_2-m_2x_1}{m_1-m_2}, \frac{m_1y_2-m_2y_1}{m_1-m_2}\right)$$



Focus Point

(i) Let $A(x_1, y_1)$ and (x_2, y_2) be the points of AB, if P(x, y) divides the line segment AB in the ratio 1 : 1 $\therefore$ $P(x, y) \equiv \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

(ii) The ratio $m_1 : m_2$ can also be written as $\dfrac{m_1}{m_2}$: 1 or k : 1 where $k = m_1 : m_2$. So, coordinates of point P(x, y) dividing line segment joining points $A(x_1, y_1)$ and $B(x_2, x_1)$ is given $(x, y) = \left(\dfrac{kx_2+x_1}{k+1}, \dfrac{ky_2+y_1}{k+1}\right)$

Example 10

Find the ratio in which point (11, 15) divides the line which joins the points (15, 5) and (9, 20).

Solution :

Let $A(x_1, y_1) = (15, 5)$, $B(x_2, y_2) = (9, 20)$ and $P(x, y) = (11, 15)$

$m_1 : m_2$ is the ratio in which P divides $A(x_1, y_1)$ and $B(x_2, y_2)$

We know that

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \text{ and } y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$\Rightarrow 11 = \frac{9m_1 + 15m_2}{m_1 + m_2} \text{ and } 15 = \frac{20m_1 + 5m_2}{m_1 + m_2}$$

$\Rightarrow 11\ (m_1 + m_2) = 9m_1 + 15m_2$ ....(i)

$15(m_1 + m_2) = 20m_1 + 5m_2$ ....(ii)

Solving either equation (i) or (ii), we will find the required ratio $\frac{m_1}{m_2} = \frac{2}{1}$.

Example 11

Find the coordinates of the point which divides the line segment joining the points (6, 3) and (–4, 5) in the ratio 3 : 2 internally.

Solution :

Let P(x, y) be the required point. Then,

$$x = \frac{3 \times (-4) + 2 + 6}{3 + 2}$$

$$\text{and } y = \frac{3 \times 5 + 2 + 3}{3 + 2}$$

$$\Rightarrow x = 0 \text{ and } y = \frac{21}{5}$$

so, the point is $\left(0, \frac{21}{5}\right)$

Example 12

Determine the ratio in which the line $3x + y - 9 = 0$ divides the line segment joining the points (1, 3) and (2, 7).

Solution :

Let the line $3x + y - 9 = 0$ divides the line segment joining at A(1, 3) and B(2, 7) in the ratio k : 1 at point C.

Then the coordinates of C are $\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$.

But C lies on $3x + y - 9 = 0$. Therefore,

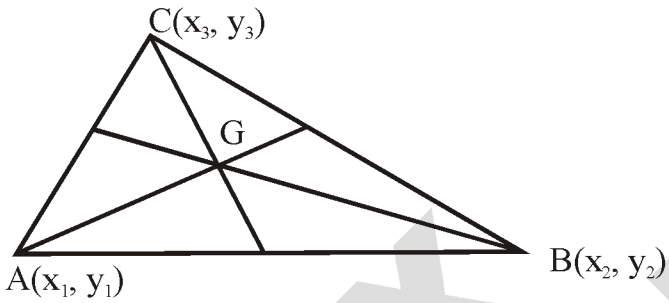
$$3\left(\frac{2k+1}{k+1}\right) + \frac{7k+3}{k+1} - 9 = 0$$

$$\Rightarrow 6k + 3 + 7k + 3 - 9k - 9 = 0 \Rightarrow k = \frac{3}{4}$$

So, the ration is 3 : 4 internally.

5. CENTROID OF A TRIANGLE

The point of intersection of medians of a triangle is called the centroid of the triangle and it divides median internally in ratio 2 : 1. Centroid is denoted by G.



If the co–ordinates of three vertices of ΔABC are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, then centroid of the triangle is

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

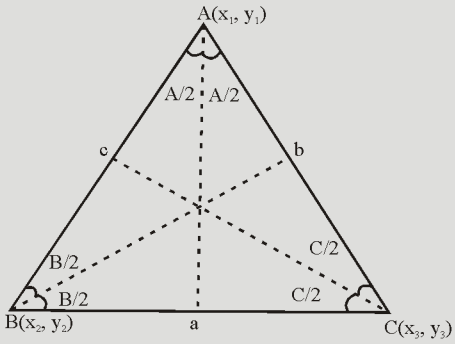
Focus Point

(i) Centroid of the triangle obtained by joining the mid points of sides of a triangle is the same as the centroid of the original triangle.

(ii) Incentre: The point of intersection of internal angle bisectors of triangle is called its incentre.

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the coordinates of the vertices of a triangle the coordinates of incentre are given by : $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$ where a, b, c are lengths of sides BC, CA and AB respectively.

(iii) If ΔABC is equilateral then a = b = c, then $I = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_3+y_3}{3}\right) = G$



i.e. incentre and centroid coincide in equilateral triangle.

Example 13

Co–ordinates of three vertices of a triangle are (4, y), (6, 9) and (x, y). Find the value of x and y, if the centroid of the triangle is (3, 6).

Solution :

Let the centroid be (h, k), then

$$h = \frac{x_1 + x_2 + x_3}{3} \text{ and } k = \frac{y_1 + y_2 + y_3}{3}$$

$$3 = \frac{4 + 6 + x}{3} \text{ and } 6 = \frac{y + 9 + 4}{3}$$

$9 = 10 + x$ and $18 = 13 + y$

$\therefore$ $x = -1$ and $y = 5$

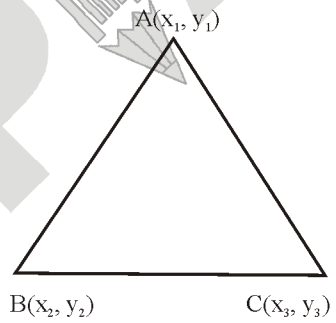
6. AREA OF A TRIANGLE

The area of a triangle, the coordinates of whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by :

$\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$ or $\frac{1}{2}|(x_1y_2 + x_2y_1 + x_3y_1) - (x_1y_3 + x_2y_1 + x_3y_2)|$. Area of a triangle is always positive.

In order to remember the formula we use following method.

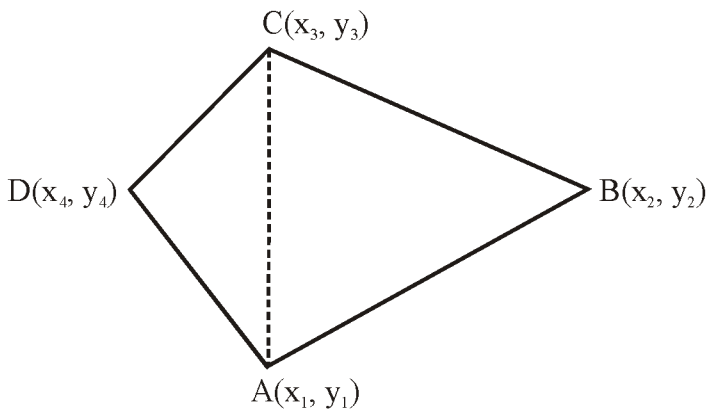
$$\begin{matrix} x_1 & & x_2 & & x_3 & & x_1 \\ y_1 & & y_2 & & y_3 & & y_1 \end{matrix}$$



7. AREA OF QUADRILATERAL

Area of quadrilateral can be found out of dividing the quadrilateral into two triangles. Let $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$ are the coordinates of vertices of a quadrilateral ABCD. Then, area of quadrilateral ABCD

$= \frac{1}{2}$ [area of ΔABC + area of ΔACD]



$$= \frac{1}{2}\,[|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| + |x_1(y_3 - y_4) + x_3(y_4 - y_1) + x_4(y_1 - y_3)|]$$

Focus Point

(i) The points $A(x_1, x_2)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear if area of $\Delta ABC = 0$ or vice–versa.

(ii) Four points $A(x_1, y_1)$, $B(x_2, y_2)$, and (x_3, y_3) are $D(x_4, y_4)$ are collinear if area of quadrilateral ABCD = 0 or vice–versa.

(iii) If one vertex (x_3, y_3) is at (0, 0) then area of $\Delta = \frac{1}{2}|x_1y_2 - x_2y_1|$.

Example 14

Find the area of the triangle, co–ordinates of whose vertices are

$(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$, and $(at_3^2, 2at_3)$

Solution :

Applying the formula of area, we have,

Required area of the triangle

$$= \frac{1}{2}\left[at_1^2.2at_2 + at_2^2.2at_3 + at_3^2.2at_1 - \left(at_2^2.2at_1 + at_3^2.2at_2. + at_1^2.2at_3\right)\right]$$

$$= \frac{1}{2}\left[2a^2\left(t_1^2t_2 + t_2^2t_3 + t_3^2t_1\right) - 2a^2\left(t_2^2t_1 + t_3^2t_2 + t_1^2t_3\right)\right]$$

$$= a^2\left[t_1t_2(t_1 - t_2) + t_2t_3(t_2 - t_3) + t_3t_1(t_3 - t_1)\right]$$

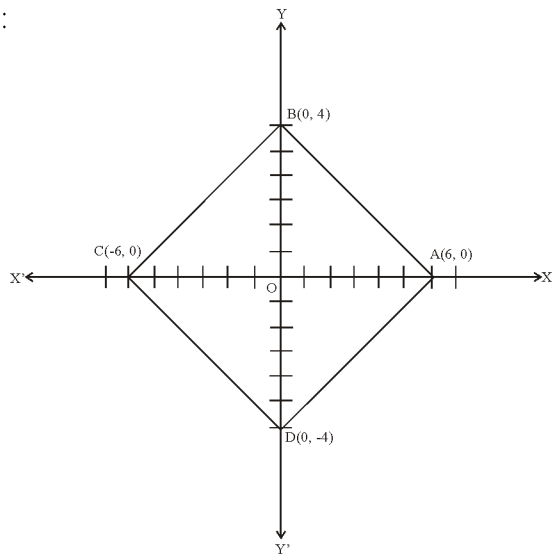
Example 15

Find the following with the help of the adjoining figure :

(a) Length of AB

(b) Mid–point of CD

(c) Area of ΔABD



Solution :

(a) $AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(6-0)^2 + (0-4)^2}$

$= \sqrt{36+16} = \sqrt{52}$

(b) Mid–point of CD is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0-6}{2}, \frac{-4+0}{2}\right) = (-3, -2).$$

(c) Area of triangle ABD $= \frac{1}{2}$ .BD. OA $= \frac{1}{2} \times 8 \times 6 = 24$

Example 16

If three points (a, b), (c, d) and (a – c, b – d) are collinear then prove that ad = bc.

Solution :

If three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear then

$$x_1 (y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$\Rightarrow a(d - b + d) + c(b - d - b) + (a - c)(b - d) = 0$

$\Rightarrow a(2d - b) + c(-d) + (a - c)(b - d) = 0 \Rightarrow 2ad - ab - cd - bc - ad + cd = 0$

$\Rightarrow ad - bc = 0 \Rightarrow ad = bc$. Hence proved.

Example 17

Show that the points (1, –1), (5, 2) and (9, 5) are collinear.

Solution : Let A(1, –1), B(5, 2) and C(9, 5) be the given points. Then,

$AB = \sqrt{(5-1)^2 + (2+1)^2} = 5$

$BC = \sqrt{(5-9)^2 + (2-5)^2} = 5$

$AC = \sqrt{(1-9)^2 + (-1-5)^2} = 10$

$\because$ AC = AB + BC

Hence, A, B, C are collinear points.

Example 18

If the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram, taken in order, find the value of p.

Solution :

Diagonals of a parallelogram bisect each other.

$\therefore$ Mid point of diagonal AC = mid point of diagonal BD.

$$\left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+p}{2}, \frac{2+3}{2}\right) \Rightarrow \frac{15}{2} = \frac{8+p}{2} \Rightarrow p = 7.$$

Example 19

Two vertices of a triangle are (3, –5) and (–7, 4). If its centroid is (2, –1), find the third vertex.

Solution :

Let the coordinates of the third vertex be (x, y)

Then, $\frac{x+3-7}{3}=2$ and $\frac{y-5+4}{3}=-1$

$\Rightarrow$ x – 4 = 6 and y – 1 = –3

$\Rightarrow$ x = 10 and y = –2

$\therefore$ third vertex is (10, –2).

Example 20

Find the incentre of the triangle whose vertices are (– 36, 7), (20, 7) and (0, – 8).

Solution :

We have, A(– 36, 7), B(20, 7) and C(0, – 8)

$a = |BC| = \sqrt{(0-20)^2+(-8-7)^2} = \sqrt{400+225} = 25$

$b = |CA| = \sqrt{(0+36)^2+(-8-7)^2} = \sqrt{1296+225} = \sqrt{1521} = 39$

$c = |AB| = \sqrt{(20+36)^2+(-7-7)^2} = \sqrt{(56)^2+0} = 56$

Now coordinates of incentre I are $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$

$\Rightarrow I = \left(\frac{25\times(-36)+39\times20+0\times56}{25+39+56}, \frac{25\times7+39\times7+56\times(-8)}{25+39+56}\right)$

$\Rightarrow I = \left(\frac{-900+780+0}{120}, \frac{175+23-448}{120}\right) \Rightarrow I(-1, 0).$

$\therefore$ coordinates of incentre of ΔABC are I (– 1, 0).

SOLVED EXAMPLES

SE. 1

In which quadrants do the following points lie :

(A) (–3, 4) (B) (4, 7)

(C) (–5, –9) (D) (6, –8)

Ans. (A) 2nd quadrant (B) 1st quadrant
(C) 3rd quadrant (D) 4th quadrant

SE. 2

A line segment is drawn joining the points, (2, 3) and (5, 3). Find the angle of this line segment with the x–axis.

Ans. The y–coordinate is equal.

Hence, this line will be parallel to the x–axis

$\therefore$ the required angle = 0°.

SE. 3

A line segment is drawn to join the points (– 4, 5) and (– 4, – 6). Find the angle of this line segment with the x–axis.

Ans. The x–coordinates are equal.

Hence, the line segment will be parallel to the y–axis.

$\therefore$ the required angle = 0.

SE. 4

A line segment is drawn to join the points (2, 5) and (–3, 4). Will this line segment cut the y–axis?

Ans. Yes, because the x–coordinates of the given points have opposite signs.

SE. 5

A line segment is drawn joining the points (–3,7) and (–5, –4). Will this line segment cut the x–axis ?

Ans. Yes, because, the y–coordinates of the points have the opposite sign.

SE. 6

There are two points (2, 5) and (7, 4). Which one will be located higher ?

Ans. (2, 5), because, its y–coordinates is more.

SE. 7

Which of the points (–3, 5) and (–5, 2) will be comparatively left ?

Ans. (–5, 2), because its x–coordinate is less.

SE. 8

Find the distance between A(2, –5) and B(–4, 2).

Ans. AB = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(2+4)^2 + (-5-2)} = \sqrt{6^2 + (-7)^2} = \sqrt{85}$.

SE. 9

Find the mid–point of line segment AB, if A is (2, –7) and B is (4, –3).

Ans. Mid points of AB = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \frac{2+4}{2}, \frac{-7-3}{2} = (3, -5)$

SE. 10

Prove that the points (2, –2), (8, 4), (5, 7) and (–1, 1) are the angular points of a rectangle.

Ans. Here A(2, –2), B(8, 4), C(5, 7) and D(–1, 1) are angular points.

AB= $\sqrt{(2-8)^2+(-2-4)^2}=\sqrt{36+36}=6\sqrt{2}$ units.

BC= $\sqrt{(8-5)^2+(4-7)^2}=\sqrt{9+9}=3\sqrt{2}$ units.

CD = $\sqrt{(5+1)^2+(7-1)^2}=6\sqrt{2}$ units.

AD = $\sqrt{(2+1)^2+(-2-1)^2}=3\sqrt{2}$ units.

AB = CD and BC = AD.

So □ ABCD may be parallelogram or rectangle

AC = $\sqrt{(2-5)^2+(-2-7)^2}=\sqrt{9+81}=\sqrt{90}$ units.

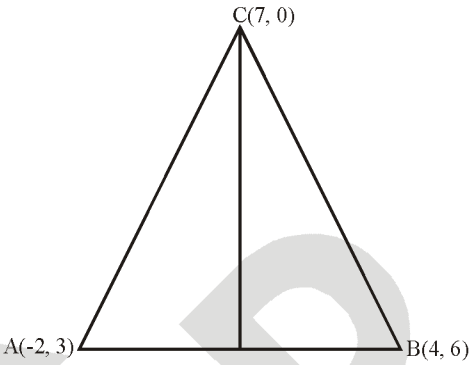
BD = $\sqrt{(8+1)^2+(4+1)^2}=\sqrt{81+9}=\sqrt{90}$ units.

AC = BD Diagonals are equal.

So ABCD must be rectangle.

SE. 11

The vertices of a triangle are (–2, 3), (4, 6) and (7, 0). Find the centroid of the triangle.



Ans. Mid–point of AB = $\left(\frac{-2+4}{2},\frac{3+6}{2}\right)=\left(1,\frac{9}{2}\right)$

Centroid divides the median in the ratio 2 : 1.

∴ The coordinates of the point dividing CD in the ratio 2 : 1 is

$$\left(\frac{2\times1+1\times7}{3},\frac{2\times\frac{9}{2}+1\times0}{3}\right)=(3,3).$$

EXERCISE – I

ONLY ONE CORRECT TYPE

1. If the coordinates of opposite vertices of a square are (1, 3) and (6, 0) the length of a side of the square is :
(A) $\sqrt{34}$ (B) $\sqrt{17}$
(C) 17 (D) 12

2. The distance between the points A(0, –1) and B(8, 3) is
(A) $4\sqrt{5}$ (B) $6\sqrt{5}$
(C) $8\sqrt{5}$ (D) none of these

3. The area of a square whose vertices are (0, – 2), (3, 1), (0, 4) and (–3, 1) is
(A) $\sqrt{18}$ sq. units (B) 18 sq. units
(C) 15 sq. units (D) $\sqrt{15}$ sq. units

4. If the distance between (8, 0) and A is 7, then coordinates of the point A can not be
(A) (8, –7) (B) (8, 7)
(C) (1, 0) (D) (0, –8)

5. If the points (0, 4), (4, 0) and (5, p) are collinear, then the value of p is
(A) –1 (B) 7
(C) 6 (D) 4

6. The ordinate of the point which divides the line joining the origin and the point (1, 2) externally in the ratio of 3 : 2 is
(A) –2 (B) $\frac{3}{5}$
(C) $\frac{2}{5}$ (D) 6

7. The vertices of a triangle are (–2, 0), (2, 3) and (1, –3), then the type of the triangle is
(A) scalene (B) equilateral
(C) isosceles (D) right angled triangle

8. The coordinates of the vertices of a rectangle are (0, 0), (4, 0), (4, 3), and (0, 3). The length of its diagonal is
(A) 4 (B) 5
(C) 7 (D) 3

9. The points which is equi–distant from the points (0, 0), (0, 8) and (4, 6) is
(A) $\left(\frac{1}{2}, -4\right)$ (B) $\left(-\frac{1}{2}, 4\right)$
(C) $\left(\frac{1}{2}, 4\right)$ (D) $\left(-\frac{1}{2}, -4\right)$

10. Points (7, 10), (–2, 5) and (3, –4) are the vertices of a/an
(A) equilateral triangle
(B) isosceles triangle
(B) right angle triangle
(D) isosceles right angled triangle

11. Three consecutive vertices of parallelogram ABCD are A(1, 2), B(1, 0) and C(4, 0). Then the fourth vertex D is
(A) (1, 2) (B) (2, 3)
(C) (3, 4) (D) (4, 2)

12. Ratio in which the point (–3, p) divides the line segment joining the points (–5, –4) and (–2, 3) is
(A) 1 : 2 (B) 2 : 3
(C) 2 : 1 (D) 3 : 2

13. The points (–3 ,2), (1, –2), (9, –10) are
(A) collinear
(B) are the vertices of triangle
(C) are in different plane
(D) none of these

14. The points (0, –1), (8, 3), (6, 7) and (–2, 3) from
(A) rectangle
(B) quadrilateral
(C) rhombus
(D) none of these

15. The points (4, –1), (6, 0), (7, 2) and (5, 1) are the vertices of a
(A) quadrilateral (B) rhombus
(C) square (D) none of these

16. Find λ, if the line $(3x - 2y + 5) + \lambda(3x - y + 4) = 0$ passes through the mid-point of the line joining the points A(2, 3) and B(4, 9).
(A) $\left(\frac{9}{10}, -\frac{7}{10}\right)$ (B) $\left(\frac{9}{10}, \frac{-7}{10}\right)$
(C) $\left(\frac{-9}{10}, -\frac{7}{10}\right)$ (D) $\left(\frac{9}{10}, \frac{7}{10}\right)$

17. Find the distance between the points (3, – 5) and (– 4, 7).
(A) $\sqrt{139}$ units (B) $\sqrt{193}$ units
(C) $\sqrt{163}$ units (D) $\sqrt{153}$ units

18. Let A(– 1, 2) and D(3, 4) be the end points of the median AD of ΔABC. Find the centroid of ΔABC.
(A) $\left(\frac{5}{3}, \frac{3}{10}\right)$ (B) $\left(\frac{5}{3}, \frac{10}{3}\right)$
(C) $\left(\frac{3}{10}, \frac{5}{3}\right)$ (D) $\left(\frac{10}{3}, \frac{5}{3}\right)$

19. Find the equation of a line passing through the point (2, – 3) and parallel to the line 2x – 3y + 8 = 0.
(A) 2x – 3y – 13 = 0 (B) 2x + 3y – 13 = 0
(C) 2x + 3y + 13 = 0 (D) 2x – 3y + 13 = 0

20. Let (– 3, 2) be one end of a diameter of a circle with centre (4, 6). Find the other end of the diameter.
(A) (10, 11) (B) (8, 10)
(C) (11, 9) (D) (11, 10)

21. Find the area of a triangle whose vertices are (a, c + a), (a, c) and (– a, c – a).
(A) a^2 (B) $a^2 + c^2$
(C) $a^2 - c^2$ (D) c^2

22. Find the co-ordinates of the point, which divides the line joining (– 3, – 5) and (4, 5) in the ratio of 4 : 3 externally.
(A) (25, 38) (B) (25, 35)
(C) (28, 35) (D) (26, 37)

23. If the points (– 2, – 1), (1, 0) (x, 3) and (1, y) form a parallelogram. Find the values of x and y.
(A) x = –4, y = 2 (B) x = –4, y = –2
(C) x = 4, y = –2 (D) x = 4, y = 2

24. If the point C(– 1, 2) divide internally the line segment joining A(2, 5) and B in ratio 3 : 4, find the coordinates of B.
(A) (– 5, – 2) (B) (5, – 2)
(C) (5, 2) (D) (– 5, 2)

25. If points (x, 0), (0, y) and (1, 1) are collinear then the relation is :
(A) x + y = 1 (B) x + y = xy
(C) x + y + 1 = 0 (D) x + y + xy = 0

PARAGRAPH TYPE

PASSAGE – I :

Let there be two points $A(x_1, y_1)$ and $B(x_2, y_2)$ in a plane. Now a point P(x, y) which divides the line segment AB in the ratio $m_1 : m_2$ internally is given by

$$P\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right).$$

26. In what ratio does the point C(–3/2, –1/4) divide the line segment joining the points A(3, 5) and B(–3, –2)?
(A) 3 : 1 (B) 1 : 3
(C) 2 : 3 (D) 3 : 2

27. Find the ratio in which the point (2, y) divides the join of (–4, 3) and (6, 3) and hence find the value of y.

(A) 2 : 3, y = 3 (B) 3 : 2, y = 4

(C) 3 : 2, y = 3 (D) 3 : 2, y = 2

28. Find the coordinates of the point which divides the line segment joining the points (6, 3) and (–4, 5) in the ratio 3 : 2 internally.

(A) (21, 0) (B) (0, 5)

(C) $\left(0, \frac{21}{5}\right)$ (D) $\left(0, \frac{5}{21}\right)$

PASSAGE – II :

Let ΔABC be any triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.

Then area of ΔABC is the numberical value of the expression

$$\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right].$$

Three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear if area of triangle formed by them is zero.

i.e., $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$.

29. If the vertices of ΔABC are (–2, 1), (5, 4) and (2, –3), then find the area of the triangle.

(A) 24 sq. units (B) 18 sq. units

(C) 20 sq. units (D) 22 sq. units

30. Find the value of k if points (k, 3) (6, –2) and (–3, 4) are collinear.

(A) –1/2 (B) –3/2

(C) 1/2 (D) 3/2

31. If A(–1, 0), B(4, 0), C(4, 3) and D(–1, 3) are the vertices of a quadrilateral ABCD. then find its area.

(A) 20 sq. units

(B) 18 sq. units

(C) 16 sq. units

(D) 15 sq. units

MATCH THE COLUMN TYPE

32. Match the Following :

Column – I	Column – II
(a) If points (k, 2k), (3k, 3k) and (3, 1) are collinear then k	(p) $k = \frac{2}{3}$
(b) Two vertex of equilateral triangle ABC are (2, 0) and (4, 0) then 3rd vertex	(q) $k = -\frac{1}{3}$
(c) If A(2, 4), B(4, 6) and C(0, 2) are vertices of ΔABC then centroid of ΔABC	(r) (2, 4)
(d) If point $C\left(\frac{3}{5}, \frac{11}{5}\right)$ divide the line segment joining the points A(3, 5), B(–3, –2) in ratio k : 1 the find k	(s) $\left(1, \sqrt{3}\right)$

(A) a→q, b→s, c→r, d→p

(B) a→s, b→q, c→r, d→p

(C) a→q, b→r, c→s, d→p

(D) a→q, b→s, c→p, d→r

33. Match the Following :

Column – I	Column – II
(a) If points (k, 5), (4, –3) and (–3, 2) are collinear, then the value of k is	(p) –2
(b) If the points A(7, –2), B(5, 1) and C(3, 2k) are collinear, then the value of k is	(q) –2/7
(c) If the points P(k, 4) lies on the line segment joining the points $A\left(-\frac{2}{5}, 6\right)$ and B(2, 9), then the value of k is	(r) 2
(d) The value of a for which the area of the triangle formed by the points A(a, a), B(2, –6) and C(3, 2) is 12 square units, is	(s) –36/5

(A) a→s, b→r, c→p, d→q

(B) a→s, b→r, c→q, d→p

(C) a→q, b→r, c→s, d→p

(D) a→s, b→q, c→p, d→r

Space for Notes :

EXERCISE – II

VERY SHORT ANSWER TYPE

1. Find the point of intersection of the lines $3x + 5y + 2 = 0$ and $4x + 7y + 3 = 0$.
2. If the point (x, y) lies in the third quadrant, then x is _____ and y is _____.
3. The line $y + 7 = 0$ is parallel to _____axis.
4. Find the point on y-axis which is equidistant from A(3, – 6) and B(– 2, 5).
5. The area of the triangle formed by the points (0, 0), (0, a), (b, 0) is _____.
6. The area of the triangle formed by the line $ax + by + c = 0$ with coordinate axes is _____.
7. The ortho-centre of the triangle formed by the points (0, 1), (1, 2) and (0, 2) is _____.
8. If the centre and radius of a circle is (3, 4) and 7, then the position of the point (5, 3) w.r.t. the circle is _____.
9. The coordinates of the points P which divides (1, 0) and (0, 0) in 1 : 2 ratio are _____.
10. If (5, 7) and (9, 3) are the ends of the diameter of a circle, then the centre of the circle is ____.

SHORT ANSWER TYPE

1. Find the length of AB, if A = (2, 4) and B(– 1, 5).
2. Prove that distance of the point $(a\cos\alpha, a\sin\alpha)$ from the origin is independent of α.
3. Using section formula show that the points (1, – 1), (2, 1) and (4, 5) are collinear.
4. Find the distance between R(a + b, a – b) S(a – b, – a – b).
5. Two vertices of a triangle are (1, 2), (3, 5) and its centroid is at the origin. Find the coordinate of its third vertex.

LONG ANSWER TYPE

1. For what value of k are the points (k, 2 – 2k), (– k + 1, 2k) and (– 4 – k, 6 – 2k) are collinear.
2. Find the midpoint of the hypotenuse of the triangle whose vertices are (4, 0), (0, 0) and (0, 6).
3. The line segment joining A(6, 3) to B(– 1, – 4) is doubled in length by having is length added to each end. Find the coordinates of new ends.
4. The vertices of a quadrilateral are (– 2, 0), (3, 0), (3, 5) and (– 2, 5). Which type of quadrilateral is it ?
5. Four points A(6, 3), B(– 3, 5), C(4, – 2) and D(x, 3x) are given in such a way that $\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$, find x.

TRUE / FALSE TYPE

1. The distance between the points (4, p) and (1, 0) is 5, if p is ± 4.
2. Straight line $\frac{x}{3} + \frac{y}{4} = 1$ passes through 1st, 3rd and 4th quadrant.
3. $\left(\frac{7}{4}, \frac{7}{8}\right)$ divide the line segment joining the points (4, –1) and (–2, 4) internally in the ratio 3 : 5.
4. Point (3,0) lies in the first quadrant.
5. Points (1,–1) and (–1,1) lie in the same uadrant.

FILL IN THE BLANKS

1. Equation of the line parallel x–axis which is five units below x–axis_____
2. The co–ordinates of the points which bisects the line joining (–3, 5) and (6, –7)_____
3. The co–ordinates of the points which divides internally joining the points (2, 3) and (5, –3) in the ratio 1 : 2_____
4. A point whose x coordinate is negative and y coordinate is positive lies in the _____ quadrant.
5. Abscissa of (3,5) is _____

ANALYTICAL PROBLEM

1. If three points (x_1, y_1), (x_2, y_2), (x_3, y_3) lie on the same line, then $\frac{y_2 - y_3}{x_2 x_3} + \frac{y_3 - y_1}{x_3 x_1} + \frac{y_1 - y_2}{x_1 x_2}$ is equal to.
2. P is the point on the y-axis which is equidistant from A(– 5, – 2) and B(3, 2), then PA = _____ cm.
3. The point P(x, y) divides the join of the points A(4, – 2) and B(– 1, 3) in the ratio 1 : 4, then find x + y.
4. If A(1, 2), B(4, y), C(x, 6) and D(3, 5) are the vertices of a parallelogram taken in order, then find xy.
5. P(x, y), A(3, 4) and B(5, – 2) are the vertices of triangle PAB such that |PA| = |PB| and area of ΔPAB = 10 square units, then PA = $k\sqrt{5}$ units. Find k.

NUMERICAL PROBLEMS

1. Distance of the point (4, a) from x-axis is double its distance from y-axis, then find a.
2. If points (3, 2), (4, k) and (5, 3) are collinear, then find value of k.
3. The coordinates of the midpoint of the line joining points (2p + 1, 4) and (5, q – 1) are (2p, q). Find the value of p + q.
4. For what positive value of x, the distance between the points (– 2, 5) and (x, 19) be $\sqrt{205}$ units ?
5. If (– 3, – 1), (a, b), (3, 3) and (4, 3) are the four vertices of a parallelogram taken in order, then find ab.

Answer Key

EXERCISE–I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
B	A	B	D	A	D	A	B	C	D	D	C	A	A	B
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
C	B	B	A	D	A	B	D	B	A	A	C	C	C	B
31	32	33												
D	A	A												

EXERCISE II

VERY SHORT ANSWER TYPE

1. (1 – 1) **2.** Negative, Negative **3.** X-axis **4.** $\left(0, \frac{-8}{11}\right)$

5. $\frac{1}{2}|ab|$ square units **6.** $\frac{1}{2}\left|\frac{c^2}{ab}\right|$

7. (0, 2) **8.** Inside the circle **9.** $\left(\frac{2}{3}, 0\right)$ **10.** (7, 5)

SHORT ANSWER TYPE

1. $\sqrt{10}$ **4.** $2\sqrt{a^2+b^2}$ **5.** (– 4, – 7)

LONG ANSWER TYPE

1. $k = \frac{1}{2}$ or $k = -1$ **2.** (2, 3) **3.** $\left(\frac{19}{2}, \frac{13}{2}\right)$ and $\left(-\frac{9}{2}, -\frac{15}{2}\right)$

4. Square **12.** $\frac{11}{8}, -\frac{3}{8}$

TRUE / FALSE

1. T **2.** F **3.** T **4.** F **5.** F

FILL IN THE BLANKS

1. y = – 5 **2.** $\left(\frac{3}{2}, -1\right)$ **3.** (3, 1) **4.** II **5.** 3

ANALYTICAL PROBLEM

1. 0 **2.** 5 **3.** 2 **4.** 18 **5.** 2

NUMERICAL PROBLEMS

1. 8 **2.** 2.5 **3.** 6 **4.** 1 **5.** 4

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : COORDINATE GEOMETRY)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

INTRODUCTION TO TRIGONOMETRY

8

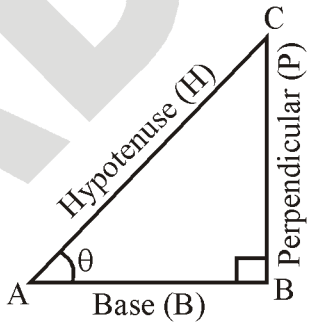
Concepts

INTRODUCTION

The word 'Trigonometry' is derived from three Greek words 'tri' means Three, 'gon' means sides, 'metron' means measure. Thus, trigonometry is the study of relationship between sides and angles of a triangle. The earliest known work on trigonometry was recorded in Egypt and Babylon. Early astronomers used to determine the distance of the stars and planets from the Earth. In broader sense it is that branch of Mathematics which deals with the measurement of the sides and angles of the triangle and the problems related to the angles.

1. TRIGONOMETRIC RATIOS

In right angle triangle, the ratio of sides of a right angled triangle is called the trigonometric ratio of angles. Let us now define various trigonometric ratios.



Let ABC be a right angled triangle with right angle at B.

$\angle A$ is an acute angle. In ΔABC, the side BC is opposite to angle A and is called the perpendicular of ΔABC. Side AC is opposite to the right angle and is known as hypotenuse of the right angle ΔABC and side AB is a part of $\angle A$, then the side AB is called the adjacent side to angle A or base of the ΔABC. There are six trigonometric ratios in the right angle triangle ABC.

(i) The ratio $\frac{BC}{AC}$ is called the sine of A and is written as sin A *i.e.*, $\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{P}{H}$

(ii) The ratio $\frac{AB}{AC}$ is called the cosine of A and is written as cos A *i.e.*, $\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{B}{H}$

(iii) The ratio $\frac{BC}{AB}$ is called the tangent of A and is written as tan A *i.e.*, $\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{P}{B}$

(iv) The ratio $\frac{AB}{BC}$ is called the cotangent of A and is written as cot A *i.e.*, $\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{B}{P}$

(v) The ratio $\frac{AC}{AB}$ is called the secant of A and is written as sec A *i.e.*, $\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{H}{B}$

(vi) The ratio $\frac{AC}{BC}$ is called the cosecant of A and is written as cosec A *i.e.*, $\operatorname{cosec} A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{H}{P}$

Focus Point

- sin A is one symbol and sin A does not mean sin × A. Similarly in the case for other trigonometric ratios.
- Every trigonometric ratio is a real number.
- In short, trigonometric ratio are written as T-ratios

1.1 RELATION BETWEEN TRIGONOMETRIC RATIOS

The trigonometric ratios as sin θ, cos θ and tan θ of an anlge θ are very closely connected by a relation. If any one of them is known, the other two can be easily calculated.

We have, $\sin\theta = \frac{P}{H}, \cos\theta = \frac{B}{H}$ and $\tan\theta = \frac{P}{B}$

Now, $\frac{\sin\theta}{\cos\theta} = \frac{P/H}{B/H} = \frac{P}{B} \quad \Rightarrow \frac{\sin\theta}{\cos\theta} = \tan\theta$

Some other relation between trigonometric ratios :

(i) $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ or $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

(ii) $\sec\theta = \frac{1}{\cos\theta}$ or $\cos\theta = \frac{1}{\sec\theta}$

(iii) $\cot\theta = \frac{1}{\tan\theta}$ or $\tan\theta = \frac{1}{\cot\theta}$

(iv) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

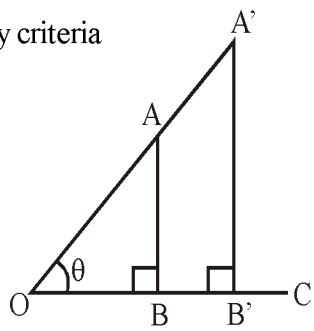
2. THEOREM

Statement : The trigonometric ratios are always same for the same angle.

Proof : We have to show that, if in the revolving line OA any other point A' can be taken and A'B' can be drawn perpendicular to OC, the ratios derived from the triangle OA'B' are same as those derived from the triangle OAB. In two triangles, ΔOAB and ΔOA'B', we have the angle at O is common and the anlges at B and B' are both right anlges and therefore are equal.

Hence, the two triangles are equiangular and therefore by AAA similarity criteria

ΔOAB ~ ΔOA'B'



$\therefore \quad \frac{OA}{OA'} = \frac{OB}{OB'} = \frac{AB}{A'B'} \quad \Rightarrow \frac{AB}{OA} = \frac{A'B'}{OA'}$

In ΔOAB and ΔOA'B', we have

$\sin\theta = \frac{AB}{OA}, \sin\theta = \frac{A'B'}{OA'}$

This shows that the value of $\sin\theta$ is independent of the position of A. Similarly, it can be proved that the other T-ratios are independent of the position of A. Therefore, the trigonometric ratios are always same for the same angle.

Focus Point

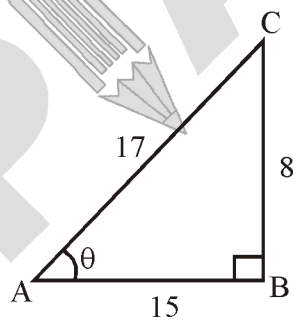
- The hypotenuse is the longest side in a right angled triangle, the value of $\sin\theta$ or $\cos\theta$ is always less than or equal to 1.

Example 1

In a ΔABC, right angled at B, if AB = 15 cm and AC = 17 cm, find all the six trigonometric ratios of angle A.

Solution :

We have AB = 15 cm and AC = 17 cm, by pythagoras theorem, we have



$AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 - AB^2$

$\Rightarrow BC^2 = (17)^2 - (15)^2$

$\Rightarrow BC^2 = 289 - 225$

$\Rightarrow BC^2 = 64$

$\Rightarrow BC = \sqrt{64} = 8$

When we consider the trigonometric ratios of $\angle A$ we have,

Base : AB = 15 ; Perpendicular : BC = 8 ; Hypotenuse : AC = 17

$\therefore \quad \sin A = \frac{P}{H} = \frac{8}{17}, \quad \cos A = \frac{B}{H} = \frac{15}{17}$

$\tan A = \frac{P}{B} = \frac{8}{15}, \quad \cot A = \frac{B}{P} = \frac{15}{8}$

$\sec A = \frac{H}{B} = \frac{17}{15}$ and $\text{cosec } A = \frac{H}{P} = \frac{17}{8}$

Example 2

If $\cos\theta = \frac{2xy}{x^2 + y^2}$, find the value of other five T-ratios.

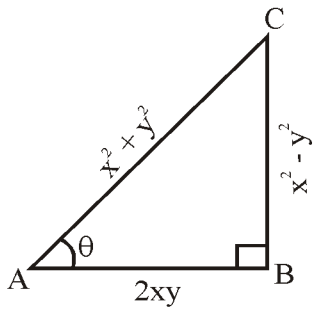
Solution :

We have, $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{2xy}{x^2 + y^2}$

So, draw a right triangle, right angled at B such that

Base = 2xy, Hypotenuse = $x^2 + y^2$ and $\angle BAC = \theta$

By Pythagoras theorem, we have



$AC^2 = AB^2 + BC^2$

$\Rightarrow (x^2 + y^2)^2 = (2xy)^2 + BC^2$

$\Rightarrow BC^2 = (x^2 + y^2)^2 - (2xy)^2$

$\Rightarrow BC^2 = x^4 + 2x^2y^2 + y^4 - 4x^2y^2$

$\Rightarrow BC^2 = x^4 - 2x^2y^2 + y^4$

$\Rightarrow BC^2 = (x^2 - y^2)^2$

$\Rightarrow BC = x^2 - y^2$

$\therefore$ Perpendicular = BC = $x^2 - y^2$

$$\therefore \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{x^2 - y^2}{x^2 + y^2}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{x^2 - y^2}{2xy}$$

$$\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{2xy}{x^2 - y^2}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{x^2 + y^2}{2xy}$$

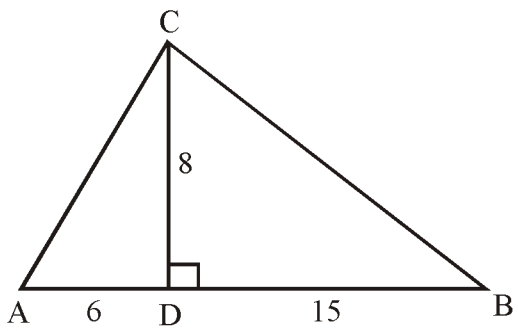
$$\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{x^2 + y^2}{x^2 - y^2}$$

Example 3

From the figure, write the value of

(i) sin A (ii) cot A (iii) tan B (iv) $\sin^2 B + \cos^2 B$

Solution :



In ΔACD, we have

$AC^2 = AD^2 + CD^2$

$\Rightarrow AC^2 = 6^2 + 8^2$

$\Rightarrow AC^2 = 36 + 64 = 100$

$\Rightarrow AC = \sqrt{100} = 10$

In ΔBCD, we have, $BC^2 = BD^2 + CD^2$

$\Rightarrow BC^2 = 15^2 + 8^2$

$\Rightarrow BC^2 = 225 + 64 = 289$

$\Rightarrow \quad BC = \sqrt{289} = 17$

(i) $\sin A = \frac{CD}{AC} = \frac{8}{10} = \frac{4}{5}$ **(ii)** $\cot A = \frac{AD}{CD} = \frac{6}{8} = \frac{3}{4}$

(iii) $\tan B = \frac{CD}{BD} = \frac{8}{15}$ **(iv)** $\sin^2 B + \cos^2 B$

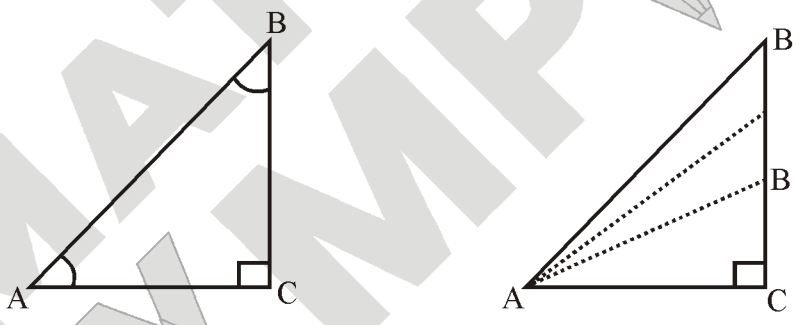
Now, $\sin B = \frac{CD}{BC} = \frac{8}{17}$ and $\cos B = \frac{BD}{BC} = \frac{15}{17}$

$\therefore \quad \sin^2 B + \cos^2 B = \left(\frac{8}{17}\right)^2 + \left(\frac{15}{17}\right)^2 = \frac{64}{289} + \frac{225}{289} = \frac{289}{289} = 1$

3. TRIGONOMETRIC RATIO OF SOME SPECIFIC ANGLES

3.1 TRIGONOMETRIC RATIO OF 0°

Consider a right angle triangle ABC right angled at C. Now ∠A is made smaller and smaller in the right ΔABC till it becomes zero. If we decrease the ∠A then the side BC also decreases and point B gets closer to the point C and finally when ∠A become 0°, the point B will coincide to C. AB becomes alomost equal to AC.



Consequently, we have

$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AB}$, then $\sin 0° = \frac{0}{AC} = 0$

Similarly, $\cos 0° = \frac{AC}{AB} = \frac{AC}{AC} = 1$, ($\because AB = AC$) and, $\tan 0° = \frac{BC}{AC} = \frac{0}{AC} = 0$,

$\cot 0° = \frac{1}{\tan 0°} = \frac{1}{0} =$ not defined, $\sec 0° = \frac{1}{\cos 0°} = \frac{1}{1} = 1$, $\text{cosec}\, 0° = \frac{1}{\sin 0°} = \frac{1}{0} =$ not defined

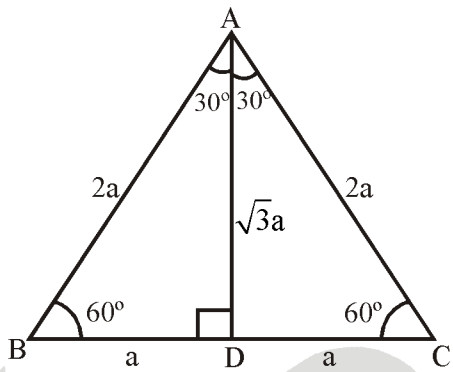
3.2 TRIGONOMETRIC RATIOS OF 30° AND 60°

Consider an equilateral ΔABC in which each side is of length 2a. Each angle of ΔABC is of 60°.

Draw AD perpendicular on BC.

Since ABC is an equilateral triangle.

$\therefore$ AD is the bisector of $\angle A$ and D is the mid point of BC



$\therefore$ $\angle BAD = 30^\circ$, $BD = CD = \frac{1}{2}BC = a$

Thus in ΔABD,

$\angle D = 90^\circ$, where $AB = 2a$, $BD = a$

$\Rightarrow \Delta ABD$ is a right angled triangle

By using Pythagoras theorem,

$AB^2 = AD^2 + BD^2$

$\Rightarrow$ $(2a)^2 = AD^2 + (a)^2$

$\Rightarrow$ $AD^2 = 4a^2 - a^2 = 3a^2$

$\Rightarrow$ $AD = \sqrt{3}a$

Now, for 30° angle :

In ΔABD, $\angle BAD = 30^\circ$, Base : $AD = \sqrt{3}a$, Perpendicular : $BD = a$, Hypotenuse : $AB = 2a$

$\therefore$ $\sin 30^\circ = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$; $\operatorname{cosec} 30^\circ = \frac{1}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2$

$\cos 30^\circ = \frac{AD}{AB} = \frac{\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$; $\sec 30^\circ = \frac{1}{\cos 30^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

$\tan 30^\circ = \frac{BD}{AD} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$; $\cot 30^\circ = \frac{1}{\tan 30^\circ} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$

Now, for 60° angle :

In ΔABD, $\angle D = 90^\circ$, $\angle B = 60^\circ$

Base : $BD = a$, Perpendicular : $AD = \sqrt{3}a$, Hypotenuse : $AB = 2a$

$\therefore \sin 60^\circ = \frac{AD}{AB} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$; $\operatorname{cosec} 60^\circ = \frac{1}{\sin 60^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

$\cos 60^\circ = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$, ; $\sec 60^\circ = \frac{1}{\cos 60^\circ} = \frac{1}{\frac{1}{2}} = 2$

$\tan 60^\circ = \frac{AD}{BD} = \frac{\sqrt{3}a}{a} = \sqrt{3}$; $\cot 60^\circ = \frac{1}{\tan 60^\circ} = \frac{1}{\sqrt{3}}$

3.3 TRIGONOMETRIC RATIOS OF 45°

Consider a right triangle ABC with right angle at B such that, $\angle A = 45°$

Hence, $\angle A + \angle C = 90°$

$\Rightarrow \quad \angle C = 45°$

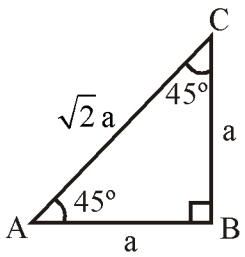
$(\because AB = BC \quad \Rightarrow \angle A = \angle C)$

Let AB = BC = a. Then by Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC^2 = a^2 + a^2 = 2a^2$

$\Rightarrow \quad AC = \sqrt{2}a$



Thus in ΔABC, $\angle A = 45°$

Base : AB = a, Perpendicular : BC = a, Hypotenuse : $AC = \sqrt{2}a$

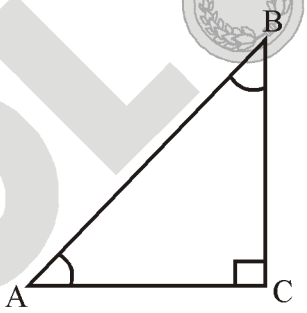
$\therefore \quad \sin 45° = \dfrac{BC}{AC} = \dfrac{a}{\sqrt{2}a} = \dfrac{1}{\sqrt{2}}, \qquad \cot 45° = \dfrac{1}{\tan 45°} = \dfrac{1}{1} = 1$

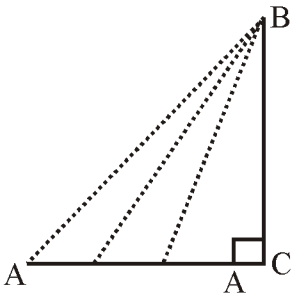
$\cos 45° = \dfrac{AB}{AC} = \dfrac{a}{\sqrt{2}a} = \dfrac{1}{\sqrt{2}}, \qquad \sec 45° = \dfrac{1}{\cos 45°} = \dfrac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

$\tan 45° = \dfrac{BC}{AB} = \dfrac{a}{a} = 1, \qquad \text{cosec } 45° = \dfrac{1}{\sin 45°} = \dfrac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

3.4 TRIGONOMETRIC RATIOS OF 90°

Consider a right triangle ABC with right angled at C. $\angle A$ of ΔABC is made large and large till it becomes 90°. If a point A move towards C, then $\angle A$ increases and side AC decreases and closer to the point C and finally $\angle A$ become 90°. The point A will coincide to C, AB almost equal to BC. So AB = BC and AC = 0.





$\therefore \quad \sin 90° = \dfrac{BC}{AB} = \dfrac{BC}{BC} = 1 \quad (\because BC = AB) \quad ; \quad \text{cosec } 90° = \dfrac{AB}{BC} = \dfrac{BC}{BC} = 1$

$\cos 90° = \dfrac{AC}{AB} = \dfrac{0}{AB} = 0, \quad ; \quad \sec 90° = \dfrac{AB}{AC} = \dfrac{AB}{0} = \text{not defined}$

$\tan 90^\circ = \frac{BC}{AC} = \frac{BC}{0} = \text{not defined}$; $\cot 0^\circ = \frac{AC}{BC} = \frac{0}{BC} = 0$

Table for T-ratio of 0°, 30°, 45°, 60°, 90°

T-Ratios \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
cot θ	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
cosec θ	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Example 4

Determine x if $\frac{x\sin^2 30^\circ \cos^2 60^\circ}{4\cos^2 45^\circ} = \frac{3\sin^2 45^\circ + 2\cos^2 30^\circ}{\sin^2 90^\circ - 4\cos^2 45^\circ}$

Solution :

$$\frac{x(\sin 30^\circ)^2(\cos 60^\circ)^2}{4(\cos 45^\circ)^2} = \frac{3(\sin 45^\circ)^2 + 2(\cos 30^\circ)^2}{(\sin 90^\circ)^2 - 4(\cos 45^\circ)^2}$$

$$\Rightarrow \frac{x\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2}{4\left(\frac{1}{\sqrt{2}}\right)^2} = \frac{3\left(\frac{1}{\sqrt{2}}\right)^2 + 2\left(\frac{\sqrt{3}}{2}\right)^2}{(1)^2 - 4\left(\frac{1}{\sqrt{2}}\right)^2} \Rightarrow \frac{x \times \frac{1}{4} \times \frac{1}{4}}{4 \times \frac{1}{2}} = \frac{3\left(\frac{1}{2}\right) + 2 \times \frac{3}{4}}{1 - 4 \times \frac{1}{2}}$$

$$\Rightarrow \frac{x}{32} = \frac{\frac{3}{2} + \frac{3}{2}}{1-2} \Rightarrow \frac{x}{32} = -3 \Rightarrow x = -96$$

Example 5

Find the value of θ in each of the following.

(i) $\tan 3\theta = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$

(ii) $\sin 2\theta = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

Solution :

(i) We have, $\tan 3\theta = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$

$\Rightarrow \tan 3\theta = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \Rightarrow \tan 3\theta = \frac{1}{2} \times \frac{1}{2} = 1 \Rightarrow \tan 3\theta = 1$

$\Rightarrow \tan 3\theta = \tan 45^\circ \Rightarrow 3\theta = 45^\circ$

$\therefore \theta = 15^\circ$

(ii) We have, $\sin 2\theta = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} \Rightarrow \sin 2\theta = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2} \Rightarrow \sin 2\theta = \frac{1}{2} \Rightarrow \sin 2\theta = \sin 30^\circ$

$\Rightarrow 2\theta = 30^\circ \Rightarrow \theta = 15^\circ$

Example 6

If $\sin (A - B) = \frac{1}{2}$ and $\cos (A + B) = \frac{1}{2}$, then find A and B.

Solution :

$\sin (A - B) = \frac{1}{2} \Rightarrow \sin (A - B) = \sin 30^\circ$

$\Rightarrow A - B = 30^\circ$ .....(1)

And, $\cos (A + B) = \frac{1}{2} \Rightarrow \cos (A + B) = \cos 60^\circ$

$\Rightarrow A + B = 60^\circ$ .....(2)

Adding (1) and (2), we get

$(A - B) + (A + B) = 30^\circ + 60^\circ$

$\Rightarrow 2A = 90^\circ \Rightarrow A = 45^\circ$

Putting $A = 45^\circ$ in (1), we get

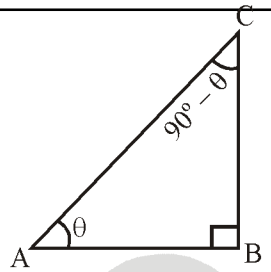
$45^\circ - B = 30^\circ$

$\Rightarrow B = 45^\circ - 30^\circ = 15^\circ$

Hence, $A = 45^\circ$ and $B = 15^\circ$

4. TRIGONOMETRIC RATIO OF COMPLEMENTARY ANGLES

4.1 COMPLEMENTARY ANGLES



Two angles are said to be complementary angles if their sum is equal to 90°.

Thus θ and (90° – θ) are complementary angles for acute angle θ.

Consider a ΔABC, right angled at B.

Let ∠CAB = θ, then ∠ACB = 90° – θ

In ΔABC, when the angle θ is considered, then AB is base and BC is perpendicular and AC is hypotenuse.

By definition of T-ratios,

$$\sin\theta = \frac{BC}{AC}, \quad \cos\theta = \frac{AB}{AC}, \quad \tan\theta = \frac{BC}{AB}, \quad \text{cosec}\,\theta = \frac{AC}{BC}, \quad \sec\theta = \frac{AC}{AB}, \quad \cot\theta = \frac{AB}{BC}$$

When the angle (90° – θ) is considered then the line BC is base, AB is perpendicular and AC is hypotenuse.

By definition of T-ratios,

$$\sin(90° - \theta) = \frac{AB}{AC} = \cos\theta, \qquad \text{cosec}(90° - \theta) = \frac{AC}{AB} = \sec\theta$$

$$\cos(90° - \theta) = \frac{BC}{AC} = \sin\theta \qquad \sec(90° - \theta) = \frac{AC}{BC} = \text{cosec}\,\theta$$

$$\tan(90° - \theta) = \frac{AB}{BC} = \cot\theta \qquad \cot(90° - \theta) = \frac{BC}{AB} = \tan\theta$$

Hence, if θ is an acute angle, then

sin (90° – θ) = cos θ, cos (90° – θ) = sin θ, tan (90° – θ) = cot θ

cosec (90° – θ) = sec θ, sec (90° – θ) = cosec θ, cot (90° – θ) = tan θ

Example 7

Evaluate :

(i) cosec (65° + θ) – sec (25° – θ) – tan (55° – θ) + cot (35° + θ)

(ii) tan 35° tan 40° tan 45° tan 50° tan 55°

Solution :

(i) We have, cosec (65° + θ) – sec (25°– θ) – tan (55° – θ) + cot (35° + θ)

= sec {90° – (65° + θ)} – sec (25° – θ) – cot {90° – (55° – θ)} + cot(35° + θ)

= sec (25° – θ) – sec(25° – θ) – cot (35° + θ) + cot (35° + θ) = 0

(ii) We have, $\tan 35^\circ \tan 40^\circ \tan 45^\circ \tan 50^\circ \tan 55^\circ$

$$= \cot(90^\circ - 35^\circ)\cot(90^\circ - 40^\circ) \times 1 \times \frac{1}{\cot 50^\circ} \times \frac{1}{\cot 55^\circ}$$

$$\Rightarrow \cot 55^\circ \times \cot 50^\circ \times \frac{1}{\cot 50^\circ} \times \frac{1}{\cot 55^\circ} = 1$$

Example 8

If $\sin 5\theta = \cos 4\theta$, where 5θ and 4θ are acute angle, find the value of θ.

Solution :

We have, $\sin 5\theta = \cos 4\theta$

$\Rightarrow \sin 5\theta = \sin(90^\circ - 4\theta)$ $\quad (\cos\theta = \sin(90^\circ - \theta))$

$\Rightarrow 5\theta = 90^\circ - 4\theta$ $\quad \Rightarrow 5\theta + 4\theta = 90^\circ$

$\Rightarrow 9\theta = 90^\circ$ $\quad \Rightarrow \theta = 10^\circ$

Example 9

If $\tan 15^\circ = 2 - \sqrt{3}$, then find the value of $\cot^2 75^\circ$

Solution :

$\tan 15^\circ = 2 - \sqrt{3}$

$\Rightarrow \cot(90^\circ - 15^\circ) = 2 - \sqrt{3} \Rightarrow \cot 75^\circ = 2 - \sqrt{3}$

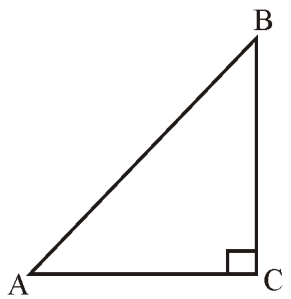
$\Rightarrow \cot^2 75^\circ = (2 - \sqrt{3})^2 = 4 + 3 - 4\sqrt{3} = 7 - 4\sqrt{3}$

5. TRIGONOMETRIC IDENTITIES

An equation involving trigonometric ratio of an angle θ is said to be an identity, if it is satisfied for all values of θ involved for which the trigonometric ratios are defined.

Here we will prove one trigonometric identity and use it further to prove other useful trigonometric identities.

Consider in ΔABC right angled at C, we have,



By pythagoras theorem,

$$AB^2 = AC^2 + BC^2$$

Dividing each term by AB^2, we get

$$\frac{AB^2}{AB^2} = \frac{AC^2}{AB^2} + \frac{BC^2}{AB^2} \quad .....(1)$$

$$\Rightarrow 1 = \left(\frac{AC}{AB}\right)^2 + \left(\frac{BC}{AB}\right)^2$$

$\Rightarrow\ 1 = (\cos A)^2 + (\sin A)^2$

$\Rightarrow\ 1 = \cos^2 A + \sin^2 A$

$\Rightarrow\ \cos^2 A + \sin^2 A = 1$

This is true for all A such that $0° \le A \le 90°$. So this is a trigonometric identity.

Let us now divide (1) by AC^2, we get

$$\frac{AB^2}{AC^2} = \frac{AC^2}{AC^2} + \frac{BC^2}{AC^2} \Rightarrow \left(\frac{AB}{AC}\right)^2 = 1 + \left(\frac{BC}{AC}\right)^2 \Rightarrow \sec^2 A = 1 + \tan^2 A$$

Similarly, when (1) is divided by BC^2, we get

$$\frac{AB^2}{BC^2} = \frac{AC^2}{BC^2} + \frac{BC^2}{BC^2} \Rightarrow \left(\frac{AB}{BC}\right)^2 = \left(\frac{AC}{BC}\right)^2 + 1 \Rightarrow \operatorname{cosec}^2 A = \cot^2 A + 1$$

Now from the above discussion, we have

(i) $\sin^2\theta + \cos^2\theta = 1$ (ii) $1 + \tan^2\theta = \sec^2\theta$ (iii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

(iv) $\sin^2\theta = 1 - \cos^2\theta$ (v) $\cos^2\theta = 1 - \sin^2\theta$ (vi) $\sec^2\theta - \tan^2\theta = 1$

(vii) $\sec^2\theta - 1 = \tan^2\theta$ (viii) $\operatorname{cosec}^2\theta - 1 = \cot^2\theta$ (ix) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

Example 10

Prove the following trigonometric identities.

(i) $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

(ii) $\dfrac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$

(iii) $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{1+\sin\theta}{\cos\theta}$

Solution :

(i) L.H.S. $= \sec^4\theta - \sec^2\theta$

$= \sec^2\theta(\sec^2\theta - 1) = (1 + \tan^2\theta)(\tan^2\theta)$ $(\sec^2\theta = 1 + \tan^2\theta)$

$= \tan^2\theta + \tan^4\theta =$ R.H.S.

(ii) L.H.S. $= \dfrac{1-\sin\theta}{1+\sin\theta}$

Multiplying and dividing by $(1 - \sin\theta)$, we get

$$= \frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)} = \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$\frac{(1-\sin\theta)^2}{\cos^2\theta} = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$= (\sec\theta - \tan\theta)^2 =$ R.H.S.

(iii) L.H.S. $= \dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$

$$= \frac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1} \qquad (1 = \sec^2\theta - \tan^2\theta)$$

$$= \frac{(\tan\theta + \sec\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1} = \frac{(\tan\theta + \sec\theta)[1 - (\sec\theta - \tan\theta)]}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta)(\tan\theta - \sec\theta + 1)}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta \qquad = \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta} = \text{R.H.S.}$$

Example 11

If $x = a \sec\theta + b \tan\theta$ and $y = a \tan\theta + b \sec\theta$. Prove that $x^2 - y^2 = a^2 - b^2$

Solution :

We have,

$x = a \sec\theta + b \tan\theta$ and $y = a \tan\theta + b \sec\theta$

Now, $x^2 - y^2$

$= (a \sec\theta + b \tan\theta)^2 - (a \tan\theta + b \sec\theta)^2$

$= (a^2 \sec^2\theta + b^2 \tan^2\theta + 2ab \sec\theta \tan\theta) - (a^2 \tan^2\theta + b^2 \sec^2\theta + 2ab \tan\theta \sec\theta)$

$= a^2 \sec^2\theta + b^2 \tan^2\theta + 2ab \sec\theta \tan\theta - a^2 \tan^2\theta - b^2 \sec^2\theta - 2ab \tan\theta \sec\theta$

$= a^2 \sec^2\theta - a^2 \tan^2\theta + b^2 \tan^2\theta - b^2 \sec^2\theta$

$= a^2(\sec^2\theta - \tan^2\theta) - b^2(\sec^2\theta - \tan^2\theta)$

$= a^2 - b^2 \qquad (1 = \sec^2\theta - \tan^2\theta = 1)$

Hence, $x^2 - y^2 = a^2 - b^2$.

SOLVED EXAMPLES

SE. 1

If $\sin A = \frac{12}{13}$, find $\cos A$ and $\tan A$.

Ans. We have, $\sin A = \frac{12}{13}$,

$\Rightarrow \quad \sin^2 A = \left(\frac{12}{13}\right)^2$

$\Rightarrow \quad 1 - \cos^2 A = \frac{144}{169} \qquad (\because \sin^2\theta = 1 - \cos^2\theta)$

$\Rightarrow \quad \cos^2 A = 1 - \frac{144}{169}$

$\Rightarrow \quad \cos^2 A = \frac{169-144}{169} \quad \Rightarrow \quad \cos^2 A = \frac{25}{169}$

$\Rightarrow \quad \cos A = \sqrt{\frac{25}{169}} = \frac{5}{13}$

We know that, $\tan A = \frac{\sin A}{\cos A} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}$

SE. 2

If $3\tan\theta = 4$, find the value of $\frac{2\sin\theta - 3\cos\theta}{2\sin\theta + 3\cos\theta}$.

Ans. We have, $\frac{2\sin\theta - 3\cos\theta}{2\sin\theta + 3\cos\theta}$

Dividing numerator and denominator by $\cos\theta$, we get

$$\frac{\frac{2\sin\theta}{\cos\theta} - \frac{3\cos\theta}{\cos\theta}}{\frac{2\sin\theta}{\cos\theta} + \frac{3\cos\theta}{\cos\theta}} = \frac{2\tan\theta - 3}{2\tan\theta + 3}$$

$$= \frac{2\left(\frac{4}{3}\right) - 3}{2\left(\frac{4}{3}\right) + 3} = \frac{\frac{8}{3} - 3}{\frac{8}{3} + 3} = \frac{\frac{-1}{3}}{\frac{17}{3}} = -\frac{1}{17}$$

(substituting $\tan\theta = \frac{4}{3}$)

SE. 3

If $\sqrt{3}\tan\theta = 3\sin\theta$. Find the value of $\sin^2\theta - \cos^2\theta$.

Ans. Here, $\sqrt{3}\tan\theta = 3\sin\theta$

$\sqrt{3}\frac{\sin\theta}{\cos\theta} = 3\sin\theta \quad \Rightarrow \quad \cos\theta = \frac{\sqrt{3}}{3}$

$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{3}} \qquad \left(\tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

$\Rightarrow \quad \cos^2\theta = \left(\frac{1}{\sqrt{3}}\right)^2 \qquad (\cos^2\theta = 1 - \sin^2\theta)$

$\Rightarrow \quad \cos^2\theta = \frac{1}{3} \quad \Rightarrow \quad 1 - \sin^2\theta = \frac{1}{3}$

$\Rightarrow \quad \sin^2\theta = 1 - \frac{1}{3} \quad \Rightarrow \quad \sin^2\theta = \frac{2}{3}$

$\therefore \quad \sin^2\theta - \cos^2\theta = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

SE. 4

Find the value of

(i) $\cos 1^\circ \cos 2^\circ \cos 3^\circ \cos 4^\circ \ldots\ldots \cos 180^\circ$

(ii) $\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \ldots\ldots + \sin^2 85^\circ + \sin^2 90^\circ$

Ans. **(i)** We have, $\cos 1^\circ \cos 2^\circ \cos 3^\circ \cos 4^\circ \ldots \cos 180^\circ$

$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 89^\circ \cos 90^\circ \cos 91^\circ \ldots \cos 180^\circ$

$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 89^\circ \times 0 \times \cos 91^\circ \ldots \cos 180^\circ = 0 \qquad [\because \cos 90^\circ = 0]$

(ii) We have, $\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \ldots + \sin^2 85^\circ + \sin^2 90^\circ$

$= (\sin^2 5^\circ + \sin^2 85^\circ) + (\sin^2 10^\circ + \sin^2 80^\circ) + \ldots + (\sin^2 40^\circ + \sin^2 50^\circ) + \sin^2 45^\circ + \sin^2 90^\circ$

$= (\sin^2 5^\circ + \cos^2 5^\circ) + (\sin^2 10^\circ + \cos^2 10^\circ) + \ldots + (\sin^2 40^\circ + \cos^2 40^\circ) + \left(\frac{1}{\sqrt{2}}\right)^2 + 1$

$\left(\sin\theta = \cos(90^\circ - \theta), \sin 45^\circ = \frac{1}{\sqrt{2}}, \sin 90^\circ = 1\right)$

$= (1 + 1 + \ldots\ldots 8 \text{ times}) + \frac{1}{2} + 1 = \frac{19}{2}$

SE. 5

Prove the following trigonometric identities.

(i) $\tan\theta + \frac{1}{\tan\theta} = \sec\theta \operatorname{cosec}\theta$

(ii) $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \tan^2\theta + \sin^2\theta$

Ans. **(i)** L.H.S. $= \tan\theta + \frac{1}{\tan\theta} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} = \operatorname{cosec}\theta\sec\theta =$ R.H.S.

(ii) L.H.S. $= (\sec\theta + \cos\theta)(\sec\theta - \cos\theta)$

$= \sec^2\theta - \cos^2\theta$

$= (1 + \tan^2\theta) - (1 - \sin^2\theta) = 1 + \tan^2\theta - 1 + \sin^2\theta$

$= \tan^2\theta + \sin^2\theta =$ R.H.S.

SE. 6

If $\sec\theta + \tan\theta = p$, show that $\frac{p^2 - 1}{p^2 + 1} = \sin\theta$

Ans. We have, $\sec\theta + \tan\theta = p$ .....(1)

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = p(\sec\theta - \tan\theta)$

$\Rightarrow \sec^2\theta - \tan^2\theta = p(\sec\theta - \tan\theta)$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p}$ .....(2)

Adding (1) and (2), we get

$\sec\theta + \tan\theta + \sec\theta - \tan\theta = p + \frac{1}{p}$

$\Rightarrow 2\sec\theta = \frac{p^2 + 1}{p}$ .....(3)

Subtracting (2) from (1), we get

$\sec\theta + \tan\theta - \sec\theta + \tan\theta = p - \frac{1}{p}$

$\Rightarrow 2\tan\theta = \frac{p^2 - 1}{p}$ .....(4)

Dividing (4) by (3), we get

$$\frac{2\tan\theta}{2\sec\theta} = \frac{\frac{p^2 - 1}{p}}{\frac{p^2 + 1}{p}} \Rightarrow \frac{\sin\theta}{\cos\theta \times \sec\theta} = \frac{p^2 - 1}{p^2 + 1}$$

$\Rightarrow$ Hence, $\sin\theta = \frac{p^2 - 1}{p^2 + 1}$

SE. 7

If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

Ans. We have, $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$\Rightarrow \sin\theta = (\sqrt{2} - 1)\cos\theta$

Multiply by $(\sqrt{2} + 1)$ on both sides, we get

$\sin\theta.(\sqrt{2} + 1) = (\sqrt{2} + 1).(\sqrt{2} - 1)\cos\theta$

$\Rightarrow \sqrt{2}\sin\theta + \sin\theta = (2 - 1)\cos\theta$

$\Rightarrow \sqrt{2}\sin\theta + \sin\theta = \cos\theta$

$\Rightarrow \cos\theta - \sin\theta = \sqrt{2}\sin\theta$

SE. 8

Prove the following identities.

(i) $\tan^2 A - \tan^2 B = \frac{\cos^2 B - \cos^2 A}{\cos^2 B \cos^2 A}$

$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$

(ii) $(1 + \tan A \tan B)^2 + (\tan A - \tan B)^2 = \sec^2 A \sec^2 B$

Ans. **(i)** We have, L.H.S. $= \tan^2 A - \tan^2 B$

$= \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B} = \frac{\sin^2 A \cos^2 B - \sin^2 B \cos^2 A}{\cos^2 A \cos^2 B}$

$= \frac{(1 - \cos^2 A)\cos^2 B - (1 - \cos^2 B)\cos^2 A}{\cos^2 A \cos^2 B}$

$= \frac{\cos^2 B - \cos^2 A \cos^2 B - \cos^2 A + \cos^2 B \cos^2 A}{\cos^2 A \cos^2 B}$

$$= \frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B} = \text{R.H.S.}$$

$$= \frac{(1-\sin^2 B)-(1-\sin^2 A)}{\cos^2 A \cos^2 B}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B} = \text{R.H.S.}$$

(ii) L.H.S. $= (1 + \tan A \tan B)^2 + (\tan A - \tan B)^2$

$= 1 + 2\tan A \tan B + \tan^2 A \tan^2 B + \tan^2 A$

$- 2\tan A \tan B + \tan^2 B$

$= 1 + \tan^2 A \tan^2 B + \tan^2 A + \tan^2 B$

$= (1 + \tan^2 A) + \tan^2 B (\tan^2 A + 1)$

$= (1 + \tan^2 A)(1 + \tan^2 B) = \sec^2 A \sec^2 B = \text{R.H.S.}$

SE. 9

If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, show that $(m^2 + n^2)\cos^2\beta = n^2$.

Ans. We have, L.H.S. $= (m^2 + n^2)\cos^2\beta$

$$= \left[\left(\frac{\cos\alpha}{\cos\beta}\right)^2 + \left(\frac{\cos\alpha}{\sin\beta}\right)^2\right]\cos^2\beta$$

$$\left[\because m = \frac{\cos\alpha}{\cos\beta}, n = \frac{\cos\alpha}{\sin\beta}\right]$$

$$= \left[\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right]\cos^2\beta$$

$$= \cos^2\alpha.\cos^2\beta\left[\frac{1}{\cos^2\beta} + \frac{1}{\sin^2\beta}\right]$$

$$= \cos^2\alpha.\cos^2\beta \times \frac{(\sin^2\beta + \cos^2\beta)}{\cos^2\beta\sin^2\beta}$$

$$= \frac{\cos^2\alpha\cos^2\beta}{\cos^2\beta\sin^2\beta} = \frac{\cos^2\alpha}{\sin^2\beta} = n^2 = \text{R.H.S.}$$

SE. 10

If $\sin\theta + \cos\theta = p$ and $\sec\theta + \text{cosec}\,\theta = q$, show that $q(p^2 - 1) = 2p$

Ans. We have L.H.S. $= q(p^2 - 1) = (\sec\theta + \text{cosec}\,\theta)$

$\{(\sin\theta + \cos\theta)^2 - 1\}$

$= (\sec\theta + \text{cosec}\,\theta)(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1)$

$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(1 + 2\sin\theta\cos\theta - 1)$$

$$= \frac{(\sin\theta + \cos\theta)}{\cos\theta\sin\theta} \times 2\sin\theta\cos\theta$$

$= 2(\sin\theta + \cos\theta) = 2p = \text{R.H.S.}$

SE. 11

If $x = a\sec\theta\cos\phi$, $y = b\sec\theta\sin\phi$ and $z = c\tan\theta$, show that $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

Ans. We have, $x = a\sec\theta\cos\phi$, $y = b\sec\theta\sin\phi$ and $z = c\tan\theta$

$$\Rightarrow \frac{x}{a} = \sec\theta\cos\phi, \frac{y}{b} = \sec\theta\sin\phi \text{ and } \frac{z}{c} = \tan\theta$$

Now, $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$

$= (\sec\theta\cos\phi)^2 + (\sec\theta\sin\phi)^2 - (\tan\theta)^2$

$= \sec^2\theta\cos^2\phi + \sec^2\theta\sin^2\phi - \tan^2\theta$

$= \sec^2\theta(\cos^2\phi + \sin^2\phi) - \tan^2\theta = \sec^2\theta - \tan^2\theta$

$= 1 = \text{R.H.S.}$

SE. 12

Find acute angle A and B, if $\sin(A+2B) = \frac{\sqrt{3}}{2}$ and $\cos(A+4B) = 0$, $A > B$.

Ans. We have, $\sin(A+2B) = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin(A+2B) = \sin 60^\circ \quad \left[\because \sin 60^\circ = \frac{\sqrt{3}}{2}\right]$

$\Rightarrow A + 2B = 60^\circ \quad .....(1)$

And $\cos(A+4B) = 0$

$\Rightarrow \cos(A+4B) = \cos 90^\circ \quad [\because \cos 90^\circ = 0]$

$\Rightarrow A + 4B = 90^\circ \quad .....(2)$

Subtracting equation (1) from equation (2), we get

$A + 4B - A - 2B = 90^\circ - 60^\circ$

$\Rightarrow 2B = 30^\circ \quad \Rightarrow B = 15^\circ$

Putting the value of B in equation (1), we get

$A + 2(15^\circ) = 60^\circ$

$\Rightarrow A + 30^\circ = 60^\circ \quad \Rightarrow A = 30^\circ$

Space for Notes :

EXERCISE – I

ONLY ONE CORRECT TYPE

1. If $\tan\theta = \frac{a}{b}$, then $\frac{a\sin\theta + b\cos\theta}{a\sin\theta - b\cos\theta}$ is equal to

(A) $\frac{a^2+b^2}{a^2-b^2}$ (B) $\frac{a^2-b^2}{a^2+b^2}$

(C) $\frac{a+b}{a-b}$ (D) $\frac{a-b}{a+b}$

2. If $3\cos\theta = 5\sin\theta$, then the value of

$\frac{5\sin\theta - 2\sec^3\theta + 2\cos\theta}{5\sin\theta + 2\sec^3\theta - 2\cos\theta}$ is

(A) $\frac{271}{979}$ (B) $\frac{316}{2937}$

(C) $\frac{542}{2937}$ (D) None of these

3. If $\frac{x\,\text{cosec}^2 30^\circ \sec^2 45^\circ}{8\cos^2 45^\circ \sin^2 60^\circ} = \tan^2 60^\circ - \tan^2 30^\circ$,

then x =

(A) 1 (B) – 1

(C) 2 (D) 0

4. If θ is an acute angle such that $\sec^2\theta = 3$, then

the value of $\frac{\tan^2\theta - \text{cosec}^2\theta}{\tan^2\theta + \text{cosec}^2\theta}$ is

(A) $\frac{4}{7}$ (B) $\frac{3}{7}$

(C) $\frac{2}{7}$ (D) $\frac{1}{7}$

5. The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ$.....$\tan 89^\circ$ is

(A) 1 (B) – 1

(C) 0 (D) None of these

6. The value of $\cos 1^\circ \cos 2^\circ \cos 3^\circ$.....$\cos 180^\circ$ is

(A) 1 (B) 0

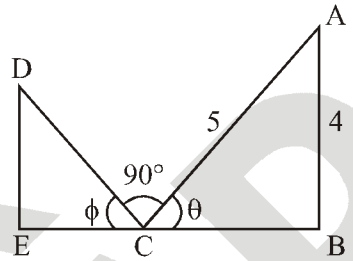
(C) – 1 (D) None of these

7. If A, B and C are interior angles of a triangle ABC, then $\sin\left(\frac{B+C}{2}\right) =$

(A) $\sin\frac{A}{2}$ (B) $\cos\frac{A}{2}$

(C) $-\sin\frac{A}{2}$ (D) $-\cos\frac{A}{2}$

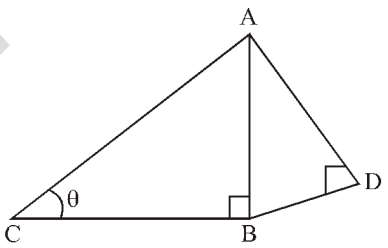
8. In Figure, the value of $\cos\phi$ is



(A) $\frac{5}{4}$ (B) $\frac{5}{3}$

(C) $\frac{3}{5}$ (D) $\frac{4}{5}$

9. In figure, if AD = 4 cm, BD = 3 cm and CB = 12 cm, then $\cot\theta =$



(A) $\frac{12}{5}$ (B) $\frac{5}{12}$

(C) $\frac{13}{12}$ (D) $\frac{12}{13}$

10. If $\cos A = \frac{5}{13}$ then what is the value of

$\frac{\sin A - \cot A}{2\tan A}$?

(A) $\frac{395}{3644}$ (B) $\frac{395}{3844}$

(C) $\frac{395}{3744}$ (D) $\frac{385}{3744}$

11. In a right angled trianlge base BC = 15 cm and sin B $= \frac{4}{5}$, then what is the length of hypotenuse AB ?
(A) 25 cm (B) 20 cm
(C) 5 cm (D) 4 cm

12. If $\sin\theta = \frac{m^2 - n^2}{m^2 + n^2}$, then value of $\tan\theta$ is
(A) $\frac{m^2 + n^2}{m^2 - n^2}$ (B) $\frac{2mn}{m^2 + n^2}$
(C) $\frac{m^2 - n^2}{2mn}$ (D) $\frac{m^2 + n^2}{2mn}$

13. If $\sin(x - y) = \frac{1}{2}$ and $\cos(x + y) = \frac{1}{2}$, then value of x is
(A) 15° (B) 30°
(C) 45° (D) 60°

14. If $1 + \tan\theta = \sqrt{2}$, then value of $\cot\theta - 1$ is
(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$
(C) 2 (D) $\frac{1}{2}$

15. If sin (x + 54°) = cosx, where 0 < x, x + 54° < 90° then what is the value of x ?
(A) 54° (B) 36°
(C) 27° (D) 18°

16. If xcos60° + ycos 0° = 3 and 4xsin 30° – ycot 45° = 2, then what is the value of x ?
(A) – 1 (B) 0
(C) 1 (D) 2

17. If $\cos x + \cos^2 x = 1$ then what is the value of $\sin^2 x + \sin^4 x$?
(A) 0 (B) 1
(C) 2 (D) 4

18. If x + y = 90° and sinx : siny = $\sqrt{3} : 1$ then x : y equals,
(A) 1 : 1 (B) 1 : 2
(C) 2 : 1 (D) 3 : 2

19. Value of
$$\frac{5\sin 75^\circ \sin 77^\circ + 2\cos 13^\circ \cos 15^\circ}{\cos 15^\circ \sin 77^\circ} - \frac{7\sin 81^\circ}{\cos 9^\circ}$$ is
(A) – 1 (B) 0
(C) 1 (D) 2

20. What is the value of $\sin^3 60^\circ \cot^2 30^\circ - 2\sec^2 45^\circ + 3\cos^2 60^\circ \tan 45^\circ - \tan^2 60^\circ$?
(A) $\frac{35}{8}$ (B) $\frac{-35}{8}$
(C) $\frac{-11}{8}$ (D) $\frac{11}{8}$

21. If $\cot\theta = \frac{2xy}{x^2 - y^2}$, then which one is equal to $\cos\theta$?
(A) $\frac{x^2 - y^2}{x^2 + y^2}$ (B) $\frac{x^2 + y^2}{x^2 - y^2}$
(C) $\frac{2xy}{x^2 + y^2}$ (D) $\frac{2xy}{\sqrt{x^2 + y^2}}$

22. For what value of θ, $(\sin\theta + \operatorname{cosec}\theta) = 2.5$, where $0 < \theta \le 90^\circ$?
(A) 30° (B) 45°
(C) 60° (D) 90°

23. In a $\triangle ABC$, $\angle ABC = 90^\circ$, $\angle ACB = 30^\circ$, AB = 5 cm. What is the length of AC ?
(A) 10 cm (B) 5 cm
(C) $5\sqrt{2}$ cm (D) $5\sqrt{3}$ cm

24. Suppose ABC is a right angled triangle with right anlged at C. If length of sides opposite to angle A, B, C are respectively u, v and w then tanA + tanB equals.
(A) $\frac{u^2}{vw}$ (B) 1
(C) u + v (D) $\frac{w^2}{uv}$

25. If A = 30° and B = 60°, then among the following which is/ are true ?
(I) sin A + sin B = cos A + cos B
(II) tan A + tan B = cot A + cot B
Use the alternative given below to get the correct answer
(A) Only I (B) Only II
(C) Both I and II (D) Neither I nor II

PARAGRAPH TYPE

PASSAGE – I : Equations like $\sin^2\theta + \cos^2\theta = 1$ and $1 + \tan^2\theta = \sec^2\theta$ which involves trigonometric ratio of an angle θ and are true for all values of θ are called trigonometric identities.

26. If $\sin\theta + \cos\theta = \sqrt{3}$, then $\tan\theta + \cot\theta$ is

(A) 1 (B) 0
(C) 2 (D) 3

27. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, then $\tan\theta$ is

(A) $\frac{1}{2}$ (B) 2
(C) – 2 (D) 3

28. If $\sec\theta - \tan\theta = \frac{1}{2}$, then the value of $\sec\theta + \tan\theta$ is

(A) 2 (B) – 2
(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

PASSAGE – II : If θ is an acute angle, then $\sin(90° - \theta) = \cos\theta$, $\cos(90° - \theta) = \sin\theta$, $\tan(90° - \theta) = \cot\theta$.

29. $\frac{\cos 70°}{\sin 20°} + \frac{\cos 55° \operatorname{cosec} 35°}{\tan 5° \tan 25° \tan 45° \tan 65° \tan 85°} =$

(A) 2 (B) 1
(C) 3 (D) 0

30. $2\left(\frac{\cos 58°}{\sin 32°}\right) - \sqrt{3}\left(\frac{\cos 38° \operatorname{cosec} 52°}{\tan 15° \tan 60° \tan 75°}\right) =$

(A) 0 (B) 2
(C) 1 (D) 3

31. $\tan 10° \tan 15° \tan 75° \tan 80° =$

(A) 0 (B) 1
(C) 2 (D) 3

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from List - I and List - II are given as options (A), (B), (C) and (D) out of which one is correct.

32. Match the following :

List – I	List – II
(P) $\frac{\sin 9°}{\cos 81°} + \sin 45°$	(i) 2
(Q) $\cos 64° - \sin 26°$	(ii) $1 + \frac{1}{\sqrt{2}}$
(R) $\sec 50° \sin 40° + \cos 40° \operatorname{cosec} 50°$	(iii) 0
(S) $\frac{\cos^3 20° - \cos^3 70°}{\sin^3 70° - \sin^3 20°}$	(iv) 1

(A) (P) → (ii), (Q) → (iv), (R) → (iii), (S) → (i)
(B) (P) → (iii), (Q) → (i), (R) → (iv), (S) → (ii)
(C) (P) → (ii), (Q) → (iii), (R) → (iv), (S) → (i)
(D) (P) → (ii), (Q) → (iii), (R) → (i), (S) → (iv)

33. In ΔABC, right angled at B and $\operatorname{cosec} A = \sqrt{2}$, then match the following :

List – I	List – II
(P) $\frac{1}{\tan A} + \frac{\sin A}{1 + \cos A}$	(i) $\sqrt{2} + 1$
(Q) $\sin C + \cos C + \sin B$	(ii) 0
(R) $\sec^2 A - \operatorname{cosec}^2 A + 2$	(iii) $\sqrt{2}$
(S) $\angle A - \angle C$	(iv) 2

(A) (P) → (iii), (Q) → (iv), (R) → (ii), (S) → (i)
(B) (P) → (iii), (Q) → (i), (R) → (iv), (S) → (ii)
(C) (P) → (ii), (Q) → (iii), (R) → (iv), (S) → (i)
(D) (P) → (iv), (Q) → (i), (R) → (ii), (S) → (iii)

EXERCISE – II

VERY SHORT ANSWER TYPE

1. Find acute angles A and B, if $\sin(A+2B) = \frac{\sqrt{3}}{2}$ and $\cos(A+4B) = 0$, $A > B$.

2. If A and B are acute angles such that $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$ and $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$, find A + B.

3. Prove that $(\sqrt{3}+1)(3 - \cot 30°) = \tan^3 60° - 2\sin 60°$

4. If $\sin(A-B) = \frac{1}{2}$, $\cos(A+B) = \frac{1}{2}$, $0° < A+B \le 90°$, $A > B$, find A and B.

5. If A, B, C, are the interior angles of a triangle ABC, Prove that $\sec\left(\frac{C+A}{2}\right) = \operatorname{cosec}\frac{B}{2}$

6. Evaluate $\frac{\tan 65°}{\cot 25°}$.

7. Without using trigonometric tables, evaluate the following :

   (i) $\frac{\cos 37°}{\sin 53°}$ (ii) $\frac{\sin 41°}{\cos 49°}$

8. Prove that :

   (i) $\sin 48° \sec 42° + \cos 48° \operatorname{cosec} 42° = 2$

   (ii) $\frac{\sin 70°}{\cos 20°} + \frac{\operatorname{cosec} 20°}{\sec 70°} - 2\cos 70° \operatorname{cosec} 20° = 0$

9. Prove the following :

   (i) $\frac{\cos(90°-\theta)\sec(90°-\theta)\tan\theta}{\operatorname{cosec}(90°-\theta)\sin(90°-\theta)\cot(90°-\theta)} + \frac{\tan(90°-\theta)}{\cot\theta} = 2$

   (ii) $\frac{\cos(90°-A)\sin(90°-A)}{\tan(90°-A)} = \sin^2 A$

   (iii) $\sin(50°+\theta) - \cos(40°-\theta) + \tan 1° \tan 10° \tan 20° \tan 70° \tan 80° \tan 89° = 1$

10. Express each one of the following terms of trigonometric ratios of angles lying between 0° and 45°

    (i) $\tan 65° + \cot 49°$

    (ii) $\cos 78° + \sec 78°$

    (iii) $\operatorname{cosec} 54° + \sin 72°$

SHORT ANSWER TYPE

1. Prove that : $(\sin^4\theta - \cos^4\theta + 1)\operatorname{cosec}^2\theta = 2$

2. Prove that : $(1 - \sin\theta + \cos\theta)^2 = 2(1+\cos\theta)(1-\sin\theta)$

3. Prove that : $(1 + \cot A + \tan A)(\sin A - \cos A) = \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A} = \sin A \tan A - \cot A \cos A$.

4. If $\sec\theta = x + \frac{1}{4x}$, then prove that : $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$

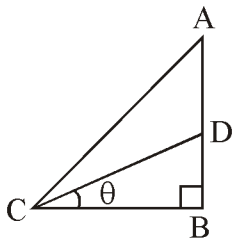
5. Prove that : $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} + \frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

   $= \frac{2}{\sin^2\theta - \cos^2\theta} = \frac{2}{2\sin^2\theta - 1} = \frac{2}{1 - 2\cos^2\theta}$

LONG ANSWER TYPE

1. If $\sin\theta = \frac{a^2-b^2}{a^2+b^2}$, find the value of $\tan\theta$.

2. In the given figure, AD = DB and ∠B is a right angle, AC = b and AB = a. Determine



   (i) $\sin\theta$ (ii) $\cos\theta$

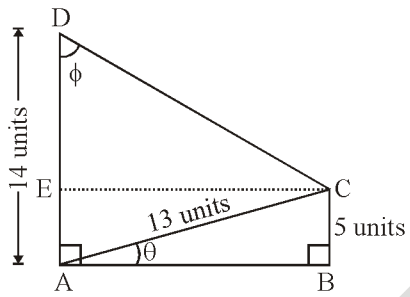
   (iii) $\tan\theta$

3. If $n \sin\theta = m \cos\theta$, then prove that

$$\frac{m\sin\theta - n\cos\theta}{m\sin\theta + n\cos\theta} + \frac{m\sin\theta + n\cos\theta}{m\sin\theta - n\cos\theta}$$

$$= \frac{2(m^4 + n^4)}{m^4 - n^4}$$

4. In the given figure, $\angle ABC = 90°$, $\angle BAC = \theta$, $\angle ADC = \phi$, BC = 5 units, AC = 13 units and AD = 14 units. Also, $\angle BAD = 90°$.



Find the value of

(i) $\cos\theta$ (ii) $\tan\phi$

(iii) $\operatorname{cosec}\phi$

5. Prove that : $\left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\operatorname{cosec}^2\theta - \sin^2\theta}\right)$

$$\times \sin^2\theta \cdot \cos^2\theta = \frac{1 - \sin^2\theta\cos^2\theta}{2 + \sin^2\theta\cos^2\theta}$$

TRUE/FALSE

1. $\tan\theta$ increases fasther than $\sin\theta$ as θ increases.
2. The value of $\sin\theta + \cos\theta$ is always less than 1.
3. $(\tan\theta + 2)(1 + 2\tan\theta) = 5\tan\theta + \sec^2\theta$.
4. The value of the expression $\cos^2 23° - \sin^2 67°$ is positive.
5. $\sqrt{(1-\cos^2\theta)\sec^2\theta} = |\tan\theta|$.

FILL IN THE BLANKS

1. Value of $(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta)$ is _ _ _ .
2. $\cot\theta$ in terms of $\cos\theta$ is_ _ _ .
3. $\cos 1°.\cos 2°.\cos 3° \ldots\ldots \cos 131° =$ _ _ _ .
4. If $\sin(A+B) = \cos(A-B) = \frac{1}{2}$, then $\operatorname{cosec} 2A =$ _ _
5. If $\alpha + \beta = 90°$ & $\beta = 2\alpha$ then $\tan\alpha \times \tan\beta =$ _ _ .

NUMERICAL PROBLEMS

1. If $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = k + \tan^2\theta + \cot^2\theta$, then find k.
2. If $0 < x \le \frac{\pi}{2}$, then $\sin x + \operatorname{cosec} x \ge$ _____.
3. If $\sin\theta = \frac{24}{25}$ and $0 < \theta < 90°$, then find $100(\sec\theta + \tan\theta)$.
4. If angles A, B, C of ΔABC are in A.P., then sec B =
5. If $\tan\theta + \cot\theta = 2$, then $\sec^2\theta - \operatorname{cosec}^2\theta =$

Answer Key

EXERCISE–I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	A	A	D	A	B	B	D	A	C	A	C	C	B	D
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
D	B	C	B	B	C	A	A	D	C	A	A	A	A	C
31	32	33												
B	D	B												

EXERCISE II

VERY SHORT ANSWER TYPE

1. $\angle A = 30°$, $\angle B = 15°$ **2.** 45° **4.** $\angle A = 45°$, $\angle B = 15°$ **6.** 1 **7.** (i) 1, (ii) 1

10. (i) cot 25° + tan 41°, (ii) sin 12° + cosec 12°, (iii) sec 36° + cos 18°

LONG ANSWER TYPE

1. $\frac{a^2 - b^2}{2ab}$ **2.** $\frac{a}{\sqrt{4b^2 - 3a^2}}, \frac{2\sqrt{b^2 - a^2}}{\sqrt{4b^2 - 3a^2}}, \frac{a}{2\sqrt{b^2 - a^2}}$ **4.** $\frac{12}{13}, \frac{4}{3}, \frac{5}{4}$

TRUE/FALSE

1. T **2.** F **3.** F **4.** F **5.** T

FILL IN THE BLANKS

1. 1 **2.** $\frac{\cos\theta}{\sqrt{1 - \cos^2\theta}}$ **3.** 0 **4.** 1 **5.** 1

NUMERICAL PROBLEMS

1. 7 **2.** 2 **3.** 700 **4.** 2 **5.** 0

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : INTRODUCTION TO TRIGONOMETRY)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

SOME APPLICATIONS OF TRIGONOMETRY

9

Concepts

Introduction

1. *Line of sight*
2. *Angle of elevation*
3. *Angle of depression*

Solved Examples

Exercise – I (Competitive Exam Pattern)

Exercise – II (Board Pattern Type)

Answer Key

INTRODUCTION

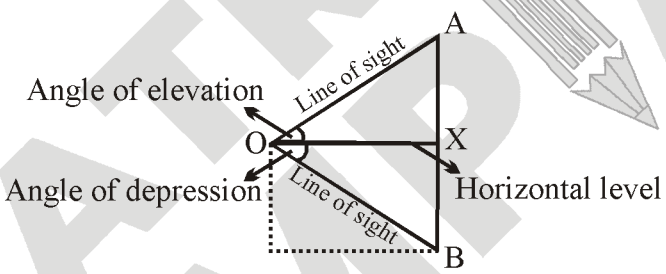
Early astronomers have used trigonometry to calculate the distances from the Earth to the planets and stars. Trigonometry is also used in geography and in navigation. Maps are constructed with the help of trigonometry to determine the position of island in relation to the longitudes and latitudes.

In this chapter, we shall applying the trigonometric results to discuss problems regarding heights and distances of various objects without measuring them.

There are some terms which will be used in this chapter.

1. LINE OF SIGHT

The line of sight is drawn from eye of an obsever to the point in the object viewed by the observer. If O is the eye of an observer and A and B are objects, then OA and OB is called the line of sight.



2. NGLE OF ELEVATION

The angle of elevation of an object viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level.

In the above given figure $\angle AOX$ is called the angle of elevation.

3. ANGLE OF DEPRESSION

The angle of depression of an object viewed is the angle formed by the line of sight with the horizontal when the point being viewed is below the horizontal level.

In the above given figure $\angle XOB$ is called the angle of depression.

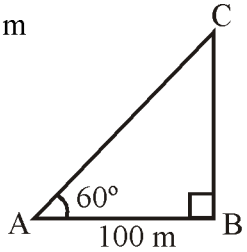
SOLVED EXAMPLES

SE. 1

Find the height of a tower if the anlge of elevation of top of tower is 60° and the horizontal distance from eye to the foot of the tower is 100 m.

Ans. Let the height of the tower be BC.

Horizontal distance AB = 100 m



In ΔABC, $\tan A = \frac{BC}{AB}$

$\therefore \quad \tan 60° = \frac{BC}{100}$

$\Rightarrow \quad \sqrt{3} = \frac{BC}{100} \quad \Rightarrow \quad BC = 100\sqrt{3}m$

Hence, height of the tower is $100\sqrt{3}$m

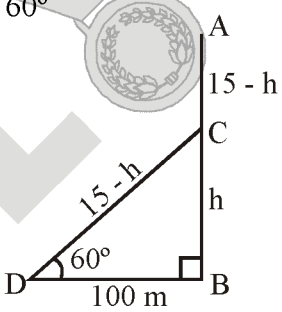
SE. 2

A vertical straight tree, 15 m high, is broken by the wind, in such a way that its top just touches the ground and makes an angle of 60° with the ground. At what height from the ground did it break ? (Use $\sqrt{3} = 1.73$)

Ans. Let the height of the tree AB = 15 m

It broke at C, its top A touches the ground at D

Now, AC = CD, $\angle BDC = 60°$



Let BC = h m

AC = AB – BC

= (15 – h)m

$\therefore \quad AC = CD = 15 - h$

In ΔBCD, $\sin 60° = \frac{h}{15-h}$

$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{h}{15-h}$

$\Rightarrow \quad \sqrt{3}(15-h) = 2h \quad \Rightarrow \quad 15\sqrt{3} - \sqrt{3}h = 2h$

$\Rightarrow \quad 2h + \sqrt{3}h = 15\sqrt{3} \quad \Rightarrow \quad h(2+\sqrt{3}) = 15\sqrt{3}$

$\Rightarrow \quad h = \frac{15\sqrt{3}}{2+\sqrt{3}} \quad \Rightarrow \quad h = \frac{15\sqrt{3}(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$

$\Rightarrow \quad h = \frac{30\sqrt{3} - 45}{4-3}$

$= 30 \times 1.73 - 45 = 51.9 - 45 = 6.9$ m

Hence, the tree broke at the height of 6.9 m from the ground.

SE. 3

From the top of a 60 m high building, the anlges of depression of the top and the bottom of a tower are observed to be 30° and 60° respectively. Find the height of the tower.

Ans. Let AB be the building and CD be the tower. Let CD = h metres. It is given that from the top of the building B, the angles of depression of the top D and the bottom C of tower CD are 30° and 60° respectively.

$\therefore \quad \angle EDB = 30°$ and $\angle ACB = 60°$

Let AC = DE = x

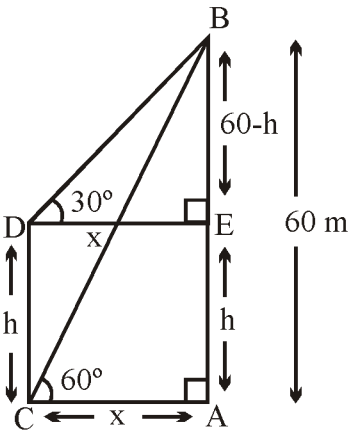
In ΔDEB, right angled at E,

we have

$\Rightarrow \tan 30° = \frac{BE}{DE}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{60-h}{x}$

$\Rightarrow \quad x = \sqrt{3}(60-h) \quad$ .....(i)



In ΔCAB, right-angled at A, we have

$$\tan 60° = \frac{AB}{CA}$$

$$\Rightarrow \quad \sqrt{3} = \frac{60}{x}$$

$$\Rightarrow \quad x = \frac{60}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$$

Putting the value of x in (1), we get

$$20\sqrt{3} = \sqrt{3}(60 - h)$$

$$\Rightarrow \quad 20 = 60 - h \qquad \Rightarrow \quad h = 60 - 20 = 40 \text{ m}$$

Thus, the height of the tower is 40 m.

SE. 4

A person standing on the bank of a river, observes that the angle of elevation of the top of a tree, standing on the opposite bank is 60°. When he moves 40 m away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and width of the river.

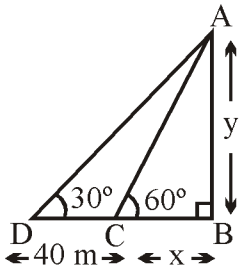
Ans. Let height of the tree AB = y metre

Width of the river CB = x metre

Let C be the point of observation and D be the other point of observation, such that CD = 40 m

In ΔABC, right angled at B,

we have $\tan 60° = \frac{AB}{BC}$



$$\Rightarrow \quad \sqrt{3} = \frac{y}{x} \quad \Rightarrow \sqrt{3}x = y \Rightarrow y = \sqrt{3}x \qquad ....(1)$$

In ΔABD, right angled at B, we have

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{y}{x+40} \qquad \Rightarrow x + 40 = \sqrt{3}y \qquad .....(2)$$

From (1) in (2), we get

$$x + 40 = \sqrt{3}(\sqrt{3}\,x)$$

$$\Rightarrow \quad x + 40 = 3x \quad \Rightarrow \quad x = 20$$

Now, putting the value of x in (1), we get

$$y = 20\sqrt{3} = 20(1.732) = 34.64$$

Hence, height of the tree (y) = 34.64 metres and width of the river (x) = 20 metres.

SE. 5

A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height h. At a point on the plane, the angle of elevation of the bottom of the flagstaff is α and that of the top of flagstaff is β. Prove that the height of the tower is $\frac{h \tan\alpha}{\tan\beta - \tan\alpha}$.

Ans. Let the height of the tower BC be x and CD be the flagstaff. Given CD = h.

Let A be the point of observation on the plane.

Let distance AB = y, ∠BAC = α and ∠BAD = β.

In ΔABC, right angled at B, we have $\tan\alpha = \frac{BC}{AB}$

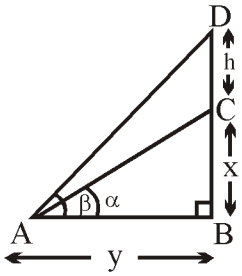
$$\Rightarrow \quad \tan\alpha = \frac{x}{y} \quad \Rightarrow \quad y = \frac{x}{\tan\alpha} \qquad ....(1)$$

In ΔABD, right angled at B, we have $\tan\beta = \frac{BD}{AB}$

$$\Rightarrow \quad \tan\beta = \frac{h+x}{y} \quad \Rightarrow \quad y = \frac{h+x}{\tan\beta} \qquad ....(2)$$

From (1) and (2), we get

$$\frac{x}{\tan\alpha} = \frac{h+x}{\tan\beta} \Rightarrow \frac{x}{\tan\alpha} = \frac{h}{\tan\beta} + \frac{x}{\tan\beta}$$



$$\Rightarrow \quad \frac{h}{\tan\beta} = \frac{x}{\tan\alpha} - \frac{x}{\tan\beta}$$

$$\Rightarrow \quad \frac{h}{\tan\beta} = x\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right)$$

$$\Rightarrow \quad \frac{h}{\tan\beta} = \frac{x(\tan\beta - \tan\alpha)}{\tan\alpha\tan\beta}$$

$$\Rightarrow \quad x = \frac{h\tan\alpha.\tan\beta}{\tan\beta(\tan\beta - \tan\alpha)}$$

$$\Rightarrow \quad x = \frac{h\tan\alpha}{\tan\beta - \tan\alpha}$$

Hence, height of the tower $= \frac{h\tan\alpha}{\tan\beta - \tan\alpha}$

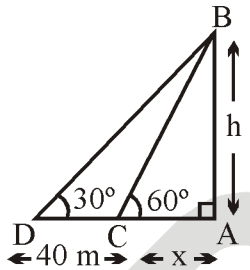
SE. 6

The shadow of a tower standing on a leveled ground is found to be 40 m longer when the sun's altitude is 30° than when it is 60°? Find the height of the tower.

Ans. Let AB be the tower and AC and AD be its shadows when the angles of elevation are 60° and 30°. Then CD = 40 metres.

Let h be the height of the tower and let AC = x m.

In ΔABC, right angled at A $\tan 60° = \frac{AB}{AC}$



$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \quad \Rightarrow \quad h = \sqrt{3}x \quad \Rightarrow \quad x = \frac{h}{\sqrt{3}} \qquad ....(1)$$

In ΔDAB, we have $\tan 30° = \frac{AB}{AD}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+40} \quad \Rightarrow \quad x + 40 = \sqrt{3}h \qquad .....(2)$$

Putting value of x from equation (1) in equation (2), we get

$$\frac{h}{\sqrt{3}} + 40 = \sqrt{3}h$$

$$\Rightarrow \quad h + 40\sqrt{3} = 3h \quad \Rightarrow \quad 2h = 40\sqrt{3}$$

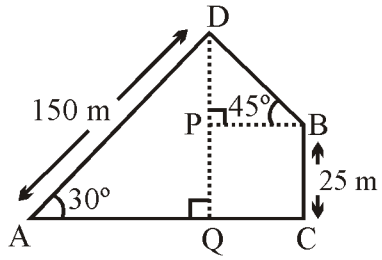
$$\Rightarrow \quad h = 20\sqrt{3}$$

Thus, the height of the tower is $20\sqrt{3}$ metre.

SE. 7

A boy is standing on the ground and flying a kite with a string of length 150 m at an angle of elevation of 30°, another boy is standing on the roof of a 25 m high building and is flying his kite at an angle of elevation 45°. Both the boys are on opposite sides of both the kites. Find the length of the string (in metres) corrects to two decimal places, that the second boy must have so that the two kites meets.

Ans. Let A be the position of first boy and D be the position of kite. Let QD be the vertical height of the kite.



AD is the string *i.e.*, AD = 150 m, $\angle DAQ = 30°$

Now in ΔAQD, right angled at Q,

$$\sin 30° = \frac{DQ}{AD} \Rightarrow \frac{1}{2} = \frac{DQ}{150} \Rightarrow DQ = \frac{150}{2} = 75$$

Let B be the position of second boy and BP be the roof. So, PQ = BC = 25

Then second kite meets the first kite at D. Then $\angle DBP = 45°$ and DP = DQ – PQ = 75 – 25 = 50 m

(DQ = 75 m)

In ΔDPB, right angled at P

$$\sin 45° = \frac{DP}{DB} \Rightarrow \frac{1}{\sqrt{2}} = \frac{50}{DB}$$

$$\Rightarrow \quad DB = 50\sqrt{2} = 50(1.4142) = 70.71$$

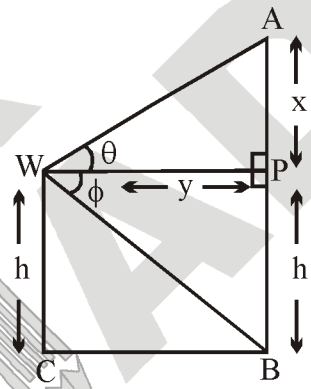
Hence, length of the string = 70.71 m

SE. 8

From a window (h metres high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are θ and ϕ respectively. Show that the height of the opposite house is $h(1 + \tan\theta \cot\phi)$

Ans. Let W be the window and AB be the house on the opposite side. Then WP is the width of the street, height of the window = h metres = BP. Let AP = x metres and WP = y metres

In ΔBPW, right angled at P, $\tan\phi = \frac{BP}{WP}$



$$\Rightarrow \quad \tan\phi = \frac{h}{y} \Rightarrow y = \frac{h}{\tan\phi} = h\cot\phi$$

Now, in ΔAPW, we have $\tan\theta = \frac{AP}{WP}$

$$\Rightarrow \quad \tan\theta = \frac{x}{y} \Rightarrow x = y\tan\theta$$

$$\Rightarrow \quad x = h\cot\phi\tan\theta \qquad [\because y = h\cot\phi]$$

Height of the opposite house = AP + BP

$= x + h = h\cot\phi\tan\theta + h = h(\cot\phi\tan\theta + 1)$

$= h(1 + \tan\theta\cot\phi)$

EXERCISE – I

ONLY ONE CORRECT TYPE

1. A flagstaff 6 metres high casts a shadow of $2\sqrt{3}$ metres long on the ground. The angle of elevation is :

(A) 30° (B) 45°

(C) 90° (D) 60°

2. An observer $\sqrt{3}$ m tall is 3 m away from the pole of $2\sqrt{3}$ m high. The angle of elevation of the top from the pole is :

(A) 45° (B) 30°

(C) 60° (D) 15°

3. An observer 1.5 m tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from her eyes is 45°. The height of the chimney is :

(A) 30 m (B) 27 m

(C) 28.5 m (D) None of these.

4. The angle of elevation of the top of a tower from a distance 100 m from its foot is 30°. The height of the tower is :

(A) $100\sqrt{3}$ m (B) $\frac{200}{\sqrt{3}}$ m

(C) $50\sqrt{3}$ m (D) $\frac{100}{\sqrt{3}}$ m.

5. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. The length of the string is :

(A) $40\sqrt{3}$ m (B) 30 m

(C) $20\sqrt{3}$ m (D) $60\sqrt{3}$ m.

6. A tree is broken by the wind. Its top struck the ground at an angle 30° at a distance of 30 m from its foot. The whole height of the tree is :

(A) $10\sqrt{3}$ m (B) $20\sqrt{3}$ m

(C) $40\sqrt{3}$ m (D) $30\sqrt{3}$ m

7. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at a height of 3 m from the banks then the width of the river is :

(A) $3\left(\sqrt{3}-1\right)$m (B) $3\left(\sqrt{3}+1\right)$m

(C) $\left(3+\sqrt{3}\right)$m (D) $\left(3-\sqrt{3}\right)$m

8. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. The height of the tower is :

(A) $\sqrt{5}$ m (B) $\sqrt{13}$ m

(C) 6 m (D) 2.25 m

9. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angles of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. The distance he walked towards the building is :

(A) $19\sqrt{3}$ m (B) $57\sqrt{3}$ m

(C) $38\sqrt{3}$ m (D) $18\sqrt{3}$ m

10. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 60°. If one ship is exactly behind the other on the same side of the light-house then the distance between the two ships is :

(A) $25\sqrt{3}$ m (B) $75\sqrt{3}$ m

(C) $50\sqrt{3}$ m (D) None of these

11. A 10 metre ladder is leaned up against a vertical wall in such a way that the mid point of the ladder is twice as far from the ground as it is from the wall. How high up on the wall does the ladder reach –

(A) $6\sqrt{2}$ m (B) $5\sqrt{3}$ m

(C) $4\sqrt{5}$ m (D) $3\sqrt{7}$ m

12. A straight highway leads to the foot of a tower of height 50 m. From the top of the tower, the angles of depression of two cars standing on the highway are 30° and 60°. What is the distance between the two cars ? (in metres)

(A) $\frac{100}{\sqrt{3}}$ (B) $50\sqrt{3}$

(C) $\frac{50}{\sqrt{3}}$ (D) $100\sqrt{3}$

13. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50 m high, what is the height of the hill ?

(A) 180 m (B) 150 m

(C) 100 m (D) 120 m

14. The angles of depression of the top and the bottom of a 7 m tall building from the top of a tower are 45° and 60° respectively. Find the height of the tower in metres.

(A) $7(3+\sqrt{3})$ (B) $\frac{7}{2}(3-\sqrt{3})$

(C) $\frac{7}{2}(3+\sqrt{3})$ (D) $7(3-\sqrt{3})$

15. There is a small island in the river which is 100 m wide and a tall tree stands on the island. P and Q are points directly opposite each other on the two banks and in line with the tree. If the angles of elevation of the top of the tree from P and Q are respectively are 30° and 45°, find the height of the tree (in metres)

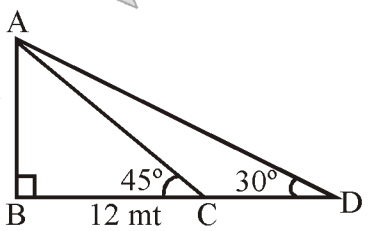
(A) $50\sqrt{3}(\sqrt{3}-1)$ (B) $50(\sqrt{3}+1)$

(C) $100(\sqrt{3}+1)$ (D) $100(\sqrt{3}-1)$

MATCH THE COLUMN TYPE

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. Only One entries of column-I may have the matching with the some entries of columnII and one entry of column-II Only one matching with entries of column-I

16. A right angle triangle ABC, point C is on BD as shown in figure.



Column-I	Column-II
(a) AB	(p) $12\sqrt{2}$ mt
(b) CD	(q) 24 mt
(c) AC	(r) 12 mt
(d) AD	(s) 8.78 mt

(A) a→s, b→r, c→p, d→q

(B) a→s, b→r, c→q, d→p

(C) a→r, b→s, c→p, d→q

(D) a→s, b→q, c→p, d→r

EXERCISE – II

VERY SHORT ANSWER TYPE

1. What is the angle of elevation of a vertical flagstaff of height $100\sqrt{3}$ m from a point 100 m from its foot.

2. A ladder makes an angle of 60° with the floor and its lower end is 20 m from the wall. Find the length of the ladder.

3. The shadow of a building is 100 m long when the angle of elevation of the sun is 60°. Find the height of the building.

4. A ladder 20 m long is placed against a vertical wall of height 10 metres. Find the distance between the foot of the ladder and the wall and also the inclination of the ladder to the horizontal.

5. What is the angle of elevation of the sun when the length of the shadow of the pole is $\frac{1}{\sqrt{3}}$ times the height of the pole ?

6. A flagstaff 6 metres high casts a shadow $2\sqrt{3}$ metres long on the ground. Find the angle of elevation of the sun..

7. A tree $10\left(2+\sqrt{3}\right)$ metres high is broken by the wind at a height $10\sqrt{3}$ metres from its root in such a way that top struck the ground at certain angle and horizontal distance from the root of the tree to the point where the top meets the ground is 10 m. Find the angle of elevation made by the top of the tree with the ground.

8. The anlge of elevation of the top of a chimney from the top of a tower is 60° and the angle of depression of the foot of the chimney from the top of the tower is 30°. If the height of the tower is 40 m, then the height of the chimney is.

9. If the angle of elevation of a cloud from a point 200 m above a lake is 30° and the angle of depression of its reflection in the lake is 60°, then the height of the cloud above the lake is.

10. Mr. Anna Hazare, Padyatra Party wanted to go from Delhi to Dehradun. The Walkers travelled 150 km straight and then took a 45° turn towards Varanasi and walked straight for another 120 km. Approximately how far was the party from the starting point.

SHORT ANSWER TYPE

1. A tree is broken at certain height and its upper part $9\sqrt{2}$ m long not completely separated meet the ground at an angle of 45°. Find the height of the tree before it was broken and also find the distance from the root of the tree to the point where the top of the tree meets the ground.

2. The ladder resting against a vertical wall is inclined at an angle of 30° to the ground. The foot of the ladder is 7.5 m from the wall. Find the length of the ladder.

3. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of vertical pole to the ground. Find the height of the pole if the angle made by the rope with the ground level is 30°.

4. The angle of depression of a ship as seen from the top of 120 m high light house is 60°. How far is the ship from the light house?

5. A man 1.8 m tall stands at a distance of 3.6 m from a lamp post and casts a shadow of 5.4 m on the ground. Find the height of the lamp post.

LONG ANSWER TYPE

1. The angle of elevation of the top of a tower from two points A and B at distance of p and q respectively from the base and in the same straight line with it, are complementary. Prove that the height of the tower is $\sqrt{pq}$.

2. From a building 60 m. high the angle of depression of the top and bottom of lamp post are 30° and 60° respectively. Find the distance between lamp post and building. Also find the difference of height between building and lamp post.

3. From an aeroplane vertically above a straight horizontal plane, the angle of depression of plane consecutive kilometre stones on the opposite side, the aeroplane are found to be 'α' and 'β'. Show that the height of the aeroplane from ground is $\frac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$

4. The angle of elevation of the top of a tower from the point on the ground is 30°. After walking 30 m. towards the tower, the angle of elevation becomes 60°. What is the height of the tower.

5. The angle of elevation of the top of tower standing on a horizontal plane from a point A is α. After walking a distance d towards the foot of the tower the angle of elevation is found to be β. Prove that the height of the tower is $\frac{d}{\cot\alpha-\cot\beta}$.

NUMERICAL ANSWER TYPE

1. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 9.5 m away from the wall. The length of the ladder is.

2. The top of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then the lenght of the wire is.

3. The angles of elevation of an artificial satellite measured from two earth stations are 30° and 60° respectively. If the distance between the earth stations is 4000 km, then the height of the satellite from ground is.

4. One side of a parallelogram is 12 cm and its area is 60 cm^2. If the angle between the adjacent side is 30°, then its other side is.

5. A tree 6 m tall cast a 4 m long shadow. At the same time, a flag pole casts a shadow 50 m long. How long is the flag pole.

Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	B	A	D	A	D	B	C	A	C	C	A	B	C	A
16														
C														

EXERCISE II

VERY SHORT ANSWER TYPE

1. 60° **2.** 40 m **3.** 173.2 m **4.** 17.32 m, $\theta = 30°$ **5.** 60° **6.** 60° **7.** 60°
8. 160 m **9.** 400 m **10.** 250 km

SHORT ANSWER TYPE

1. $9\left(\sqrt{2}+1\right)$m , 9 m **2.** 8.66 m **3.** 10 m **4.** 69.28 m **5.** 3 m

LONG ANSWER TYPE

2. $20\sqrt{3}$ m, 20 m **4.** $15\sqrt{3}$ m

NUMERICAL PROBLEMS

1. 19 m **2.** 12 m **3.** 3464 km **4.** 10 cm **5.** 75 m

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : SOME APPLICATIONS OF TRIGONOMETRY)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

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